Contra- Semi -Pre-Continuous Functions

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Abstract

In this paper, we introduce and investigate contra-semi-pre-continuous functions. This new class is a super class of the class of contra-α –continuous functions and contra-pre-continuous functions.

Key word : **Contra- Semi -Pre-Continuous Functions.**

الدوال المستمرة العكسية من نمط شبه قبلي

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الخلاصة

في هذا البحث تم تقديم والتطرق للدوال المستمرة العكسية من نمط شبه قبلي وهذه الفئة الجديدة هي فئة عظمى من فئة الدوال المستمرة العكسية نوع – α والدوال المستمرة العكسية قبليا.

الكلمات الدالة: **Contra- Semi -Pre-Continuous Functions**

1.Introduction

Dontchev [1] introduced the notion of contra-continuity and obtained some results concerning compactness, S-closedness and strong S-closedness in 1996.Dontchev and Noiri [2] introduced and investigated contra-semi-continuous functions and RC-continuous functions between topological spaces in 1999. Jafari and Noiri [3] introduced contra-pre-continuous functions and obtained their basic properties. Jafari and Noiri [4] also introduced contra-α-continuous functions between topological spaces. Recently Veera Kumar [5] introduced the class of contra-ψ-continuous functions. Recently Veera Kumar [6] introduced pre-semi-closed sets for topological spaces. In this paper, we introduce and investigate contra-semi-pre-continuous functions. This new class is a superclass of the class of contra-α -continuous functions and contra-pre-continuous functions.

2.Preliminaries

Throughout this paper,(X,τ) and (Y,σ) will denote topological spaces. For a subset A of a space (X,τ) ,cl(A) (resp. int(A) and C(A) will denote the closure (resp. the interior and the complement) of A in (X, τ).

Definition 2.1. A subset A of a topological space (X, τ) is called

(1) semi-open if $A \subseteq cl(\text{int}(A))$ [7]

(2) pre-open if $A \subseteq \text{int}(cl(A))$ [8]

(3) α-open if $A \subseteq (\text{int}(cl(\text{int}(A)))$ [9]

(4) β -open [10] or semi-pre-open [11] if $A \subseteq cl(\text{int}(cl(A)))$

(5) regular open if $A = \text{int}(cl(A))$ [7]

(6) regular closed if $A = cl(\text{int}(A))$ [7]

The complement of a pre open (resp. semi-open, α-open, β -open) set is called a pre closed (resp. semi-closed, α-closed, β -closed) set.

Definition 2.2. A subset A of a space (X, τ) is called a generalized closed (briefly g-closed) set [12] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ). The complement of a g-closed set is called a g-open set.

Definition 2.3. A subset A of a space (X, τ) is called a semi-pre-closed set [6] if $PSCL(A) \subseteq U$ whenever $A \subseteq U$ and U is a g-open set of (X, τ), where $pscl(A)$ is the pre-semi-closure of A. The complement of a semi-pre-closed set A is called a semi-pre-open set.

Definition 2.4. A subset A of a space (X, τ) is called a semi-generalized closed (briefly sg-closed) set [11] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ), where $scl(A)$ is the semi-closure of A. The complement of a sg-closed set is called a sg-open set.

Definition 2.5. A subset A of a space (X, τ) is called a ψ-closed set [13] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is a sg -open set of (X, τ). The complement of a ψ-closed set A is called a ψ-open set.

Definition 2.6. A function $f:(X,\tau) \to (Y,\sigma)$ is called:

(1) perfectly continuous [14] or strongly continuous [10] if $f^{-1}(V)$ is clopen in(X, τ) for every open set V of (Y, σ)

(2) RC-continuous [2] if $f^{-1}(V)$ is regular closed in (X, τ) for every open set V of (Y, σ)

(3) contra-continuous [1] if $f^{-1}(V)$ is closed in (X, τ) for every open set V of (Y, σ)

(4) contra-pre-continuous [3] if $f^{-1}(V)$ is pre-closed in (X, τ) for every open set V of (Y, σ)

(5)contra-semi-continuous[2] if $f^{-1}(V)$ is semi-closed in (X, τ) for every open set V of (Y, σ)

(6) contra-α-continuous [4] if $f^{-1}(V)$ is α-closed in (X, τ) for every open set V of (Y, σ)

(7) contra-ψ-continuous [5] if $f^{-1}(V)$ is ψ-closed in (X, τ) for every open set V of (Y, σ)

Definition 2.7. A topological space(X, τ) is called:

(1) a semi-pre-$T_{1/2}$ space [6] if every semi-pre-closed set in it is pre- semi-closed

(2) a semi-pre- T_b space [6] if every semi-pre-closed set in it is pre-closed

(3) a semi-pre-$T_{3/4}$ space [15] if every semi-pre-closed set in it is semi-closed.

3. Contra- semi - pre -continuous functions

Definition 3.1. A function $f:(X,\tau)\to(Y,\sigma)$ is called contra- semi-pre-continuous if $f^{-1}(V)$ is semi-pre-closed in (X, τ) for each open set V of (Y, σ).

Theorem 3.1. Every contra-α-continuous function is contra-semi-pre-continuous.

Proof. It follows from the fact that every pre-semi-closed set is semi-pre-closed ([15,Theorem 3.02]).

Example 3.1. A contra-semi-pre-continuous function need not be contra-α-continuous. Let $X = Y = \{a,b,c\}, \tau = \{\emptyset, X, \{a\}, \{a,c\}\}$ and $\sigma = \{\emptyset, Y, \{a\}\}$. Define $f:(X,\tau)\to(Y,\sigma)$ by $f(a) = a, f(b) = a$ and $f(c) = b$. Then f is contra-semi-pre-continuous but not contra- α-continuous.

Thus the class of contra-semi-pre-continuous functions properly contains the class of contra-α-continuous functions.

Theorem 3.2. Every contra-semi-continuous function is contra-semi-pre-continuous.

Proof. It follows from the fact that every semi closed set is semi-pre-closed ([15,Theorem 3.04]).

Example 3.2. A contra-semi-pre-continuous function need not be contra-semi-continuous. Let $X = Y = \{a,b,c\}, \tau = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}\}$ and $\sigma = \{\emptyset, Y, \{a\}\}$.

Define $g : (X, \tau) \rightarrow (Y, \sigma)$ by $g(a) = a, g(b) = b$ and $g(c) = c$. Then g is contra-semi-pre-continuous but not contra-semi-continuous.

Thus the class of contra-semi-pre-continuous functions properly contains the class of contra-semi-continuous functions.

Definition 3.2. A space (X, τ) is called semi-pre-locally indiscrete if every semi-pre-open set in it is closed.

Example 3.3. Let X = {a, b} and τ = {∅, X, {a}, {b}}. Then (X, τ) is a semi-pre-locally indiscrete space. The space (X, τ) in Example 3.2 is not a semi-pre-locally indiscrete space since {a} is a semi-pre-open set but it not closed.

Theorem 3.3. If a function $f : (X, \tau) \rightarrow (Y, \sigma)$ is semi-pre-continuous and (X, τ)is semi-pre-locally indiscrete, then f is contra-continuous.

Proof. Let V be an open set of (Y, σ) Then $f^{-1}(V)$ is semi-pre-open in (X, τ)

since f is semi-pre-continuous. Since (X, τ) is semi-pre-locally indiscrete, $f^{-1}(V)$ is closed. Therefore f is contra-continuous.

Theorem 3.4. If a function $f : (X, \tau) \rightarrow (Y, \sigma)$ is contra-semi-pre-continuous and(X, τ) is a semi-pre- $T_{1/2}$ space, then f is contra- α-continuous.

Proof. Let V be an open set of (Y, σ) Then $f^{-1}(V)$ is semi-pre-closed in(X, τ)

since f is contra-pre-semi-continuous. Since (X, τ) is semi-pre-$T_{1/2}$, $f^{-1}(V)$ is pre-semi-closed. Therefore f is contra-α-continuous.

Theorem 3.5. If a function $f : (X, \tau) \rightarrow (Y, \sigma)$ is contra-semi-pre-continuous and (X, τ) is a semi-pre-T_b space, then f is contra-pre-continuous.

Proof. Let V be an open set of (Y, σ) Then $f^{-1}(V)$ is semi-pre-closed in (X, τ)since f is contra-semi-pre-continuous. Since (X, τ) is pre-semi-T_b , $f^{-1}(V)$ is pre-closed. Therefore f is contra-pre-continuous.

Theorem 3.6. If a function$f : (X, \tau) \rightarrow (Y, \sigma)$ is contra-semi-pre-continuous and(X, τ)is a semi-pre-$T_{3/4}$ space, then f is contra-semi-continuous.

Proof. Let V be an open set of (Y, σ). Then $f^{-1}(V)$ is semi-pre-closed in (X, τ) since f is contra-semi-continuous. Since(X, τ) is semi-pre-$T_{3/4}$, $f^{-1}(V)$ is semi closed. Therefore f is contra-pre-continuous.

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