Journal of Kirkuk University –Scientific Studies , vol.2, No.1,2007

On Zero-Symmetric Left Centrally Prime Near-Rings

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Abstract

Our aim in this paper is: to give some properties of zero-symmetric left centrally prime near-rings, then looking for those conditions which make zero-symmetric left centrally prime near-rings abelian, so that several conditions are given under which zero-symmetric left centrally prime near-rings become abelian.

Introduction

A left near-ring is a set N together with two binary operations (+) and (.) such that:

i: $(N,+)$ is a group (not necessarily abelian).

ii: $(N,.)$ is a semigroup.

iii: for all $a,b,c \in N, a.(b+c) = a.b + a.c$.(left distributive law).(Pilz, 1983).

Let N be a left near-ring then:

N is called a left prime near-ring if for $x, y \in N, xNy = \{0\}$ implies $x = 0$ or $y = 0$, (Argac,2004), and it is called a zero-symmetric left near-ring if $0a = 0,$ for all $a \in N$ (Pilz, 1983), also N is said to be a left abelian near-ring if $(N,+)$ is an abelian group, (Kandasamy,2002).An additive mapping $D: N \to N$ is called a derivation on N if $D(ab) = D(a)b + aD(b),$ for all $a,b \in N$ (Wang, 1994), and a derivation $D: N \to N$ is called a Daif $1-$derivation if for all $x, y \in N$ we have $D([x, y]) = xy - yx,$ (Bell and Mason, 1992), and it is called a Daif $2-$derivation if $D([x, y]) = -xy + yx,$ for all $x, y \in N$ (Deng,Yenigul and Argac, 1998).Finally if S is a multiplicative system in N then we say S has zero commutator if $[S,N] = \{0\}$, where $[S,N] = \{[s,a]: s \in S, a \in N\}$, and $Z(N)$ will denote the center of N .Finally N is called a 2-torsion free if for $x \in N$, $2x = 0$ implies that $x = 0$ (Vukman, 2001).

Section 1: Construction of the left near-ring N_S

In this section we try to localize zero-symmetric left near-ring at multiplicative systems which have zero commutators.

Let N be a zero-symmetric left near-ring and S a multiplicative system in N such that $[S,N]=\{0\}$.We define a relation (~) on $S\times N$ as follows: for $(a,s),(b,t)\in N\times S,(a,s)\sim(b,t)$ if and only if there exists $k\in S$ such that
$k(at-bs)=0$, or equivalently $(at-bs)k=0$, (since $[S,N]=\{0\}$).It can be show that (~) is an equivalence relation on $N\times S$ .Now let us denote the equivalence class of (a,s) in $N\times S$ by a_S, and the set of all equivalence classes determined by this quivalence relation by N_S ,that is $a_S=\{(b,t)\in N\times S:(a,s)\sim(b,t)\}$ and $N_S=\{a_s:(a,s)\in N\times S\}$.On N_S we define addition $(+)$ and multiplication $(.)$ as:

$a_s+b_t=(at+bs)_{st}$, and $a_s.b_t=(ab)_{st}$, for all $a_s,b_t\in N_S$.

It can be shown that the addition and multiplication as defined above are well-defined and $(N_S,+,.)$ forms a left near-ring.

Next we mention that all rings under consideration are with non zero center $Z(R)$, and the proofs of the followibg theorems can be found in the indicated references.

Theorem (A): (Cho, 2001)

Let D be a derivation on on a zero-symmetric left prime near-ring N and let $a\in N$.If for all $x\in N$ we have $aD(x)=0\,(or\ D(x)a=0)$, then either $a=0$ or $D=0$.

Theorem (B): (Cho, 2001)

Let N be a zero-symmetric left prime near-ring with nonzero derivations D_1 and D_2 on N such that $D_1(x)D_2(y)=-D_2(x)D_1(y)$, for all $x,y\in N$, then N is an abelian near-ring.

Theorem (C): (Cho, 2001)

Let N be a zero-symmetric left prime near-ring of $2-$torsion free and let D_1 and D_2 be derivations on N with the condition $D_1(a)D_2(b)=D_2(b)D_1(a)$, for all $a,b\in N$, then D_1D_2 is a derivation on N if and only if either $D_1=0$ or $D_2=0$.

Theorem (D): (Cho, 2001)

Let N be a zero-symmetric left prime near-ring of $2-$torsion free , and let D be a derivation on N such that $D^2 = 0$ then $D = 0$.

Theorem (E): (Deng,Yenigul and Argac, 1998)

Let n be a positive integer and N an $n-$torsion free zero-symmetric left prime near-ring and $D: N \to N$ is a derivation on N with $D^n(N) \subseteq Z(N)$.If $D(Z(N)) \neq \{0\}$ then $D^n(Z(N)) \neq \{0\}$ and $(N,+)$ is an abelian group.

Theorem (F): (Bell & Mason, 1992)

If N is a zero symmetric left prime near-ring admitting a nonzero Daif $1-$derivation, then $(N,+)$ is abelian, and if N is $2-$torsion free, then N is a commutative ring.(Note that a zero-symmetric left near-ring N is called abelian if the addition (+) is commutative and it is called commutative if the multiplication (.)is commutative).

Now we introduce the following definitions:

Definitions

Let N be a left near-ring.We say N is a left centrally prime near-ring if N_S is a left prime near-ring for each multiplicative system S in N with $[S,N] = \{0\}$.

Also we say a derivation $D: N \to N$ is a centrally zero derivation if $D(S) = \{0\}$ for each multiplicative system S in N with $[S,N] = \{0\}$.

Before giving the main results of this paper we prove some propositions which will lead to the proofs of the main theorems.

Prop. (1):

If N is a left near-ring with $D: N \to N$ is a centrally-zero derivation then $D_*: N_S \to N_S$ defined by $D_*(a_S) = (D(a))_S$, for all $a_S \in N_S$, is a derivation on N_S.(We call the derivation D_* the induced derivation by D).

Proof:

For all $a_S \in N_S$, where $a \in N, s \in S$, we have $D(a) \in N$ and hence $D_*(a_S) = (D(a))_S \in N_S$ and if $a_S = b_t \in N_S$, where $a,b \in N, s,t \in S$, there exists $u \in S$ such that $u(at - bs) = 0$ or $uat = ubs$.Then $atu = bsu$ (since $[S,N] = \{0\}$). Hence $D(atu) = D(bsu)$ or $D(a)tu = D(b)su$ .So $u(D(a)t) = u(D(b)s)$. Hence $u(D(a)t - D(b)s) = 0$ which means

$(D(a))_s = (D(b))_t$.Hence $D_*(a_s) = D_*(b_t)$ and so D_* is a mapping.Next we have
$D_*(a_s + b_t) = D_*((at + bs)_{st}) = (D(at + bs))_{st} = (D(a)t + D(b)s)_{st} = (D(a)t)_{st} + (D(b)s)_{ts} = (D(a))_s t_t + (D(b))_t s_s = (D(a))_s + (D(b))_t = D_*(a_s) + D_*(b_t)$ and also $D_*(a_s b_t) = D_*((ab)_{st}) = (D(ab))_{st} = (aD(b))_{st} + (D(a)b)_{st} =$
$a_s (D(b))_t + (D(a))_s b_t = a_s D_*(b_t) + D_*(a_s) b_t$.
Hence D_* is a derivation on N_S ♦.

Prop. (2):

Let N be a left near-ring and S a multiplicative system in N such that $[S, N] = \{0\}$.If n is a positive integer such that N is an $n-$torsion free then so is N_S.

Proof:

Since N is a left near-ring so N_S is a left near-ring.To show N_S is an $n-$torsion free.Let $na_s = 0$, for $a_s \in N_S$ then $a_s + a_s + a_s + ... + a_s = 0$ ($n$times)or $(a + a + a + ... + a)_s = 0$ (n times).
Thus there exists $t \in S$ such that $t(a + a + a + ... + a) = 0$.
Hence $ta + ta + ... + ta = 0$, that is $n(ta) = 0$ and N being $n-$torsion free, we get $ta = 0$.Then $a_s = t_t a_s = (ta)_{ts} = 0_{ts} = 0$.
Hence N_S is an $n-$torsion free left near-ring ♦.

Prop. (3):

Let N be a left near-ring and S a multiplicative system in N with $[S, N] = \{0\}$.
If $D : N \to N$ is a centrally zero Daif 1-derivation (resp.a centrally zero Daif 2-derivation) on N then so is the induced derivation $D_* : N_S \to N_S$.

Proof:

Let D be a Daif $1-$derivation.Then by **Prop. (1)**, D_* is a derivation on N_S.

Now we show D_* is a Daif 1-derivation.Let $a_s, b_t \in N_S$, where $a,b \in N$ and $s,t \in S$, then $D_*([a_s, b_t]) = D_*(([a,b])_{st}) = (D([a,b]))_{st} = (ab - ba)_{st} = (ab)_{st} - (ba)_{ts} = a_s b_t - b_t a_s$ and thus D_* is a Daif 1-derivation.If D is a Daif $2-$derivation then a similar argument to prove that D_* is a Daif $2-$derivation ♦.

Prop. (4):

Let N be a left near-ring such that $Z(N)$ has no proper zero divisors and S is a multiplicative system in N.Then $S = Z(N) - \{0\}$ is a multiplicative system in N.

Proof:

Clearly $0 \notin S$.Let $a,b \in S$.So $a,b \in Z(N)$ and $a \neq 0, b \neq 0$. Hence $ab \in Z(N)$.

Now if $ab = 0$.As $Z(N)$ contains no proper zero divisors we get $a = 0$ or $b = 0$

which is a contradiction.Thus $ab \neq 0$ which, in consequence, implies that $ab \in Z(N) - \{0\} = S$.Hence $S = Z(N) - \{0\}$ is a multiplicative system in N ♦.

Prop. (5):

If N is a zero-symmetric left near-ring and S is a multiplicative system in N such that $[S,N] = \{0\}$ then N_S is also a zero-symmetric left near-ring.

Proof:

The proof is simple.

Prop. (6):

Let N be a left near-ring and S a multiplicative system in N with $[S,N] = \{0\}$. Then $(Z(N))_S \subseteq Z(N_S)$.

Proof:

Note that $Z(N)$ is a subnear-ring of N and $S \subseteq Z(N)$ so we can consider S as a multiplicative system in the near-ring $Z(N)$ which alows talking about $(Z(N))_S$.

If $a_s \in (Z(N))_S$,where $a \in Z(N), s \in S$.To show $a_s \in Z(N_S)$.

Let $x_t \in N_S$, where $x \in N, t \in S$.Then we have $[a,x] = 0$.

Hence $[a_s, x_t] = a_s x_t - x_t a_s = (ax - xa)_{st} = ([a,x])_{st} = 0_{st} = 0$ which means $[a_s, N_S] = \{0\}$.That is $a_s \in Z(N_S)$.So that $(Z(N))_S \subseteq Z(N_S)$ ♦.

Section 2: Some properties of zero symmetric left centrally prime near-rings

Now we give some properties of zero-symmetric left centrally prime near-rings.

Theorem (1):

Let N be a zero-symmetric left centrally prime near-ring in which $Z(N)$ has no proper zero divisors and $D : N \to N$ be a centrally zero derivation on N and $a \in N$.If $aD(x) = 0 (or\ D(x)a = 0),$ for all all $x \in N$, then either $a = 0$ or $D = 0$.

Proof:

By **Prop. (4),** $S = Z(N) - \{0\}$ is a multiplicative system in N, where $[S, N] = \{0\}$, and by **Prop. (5),** N_S is zero-symmetric.Now consider the induced derivation $D_* : N_S \to N_S$, on N_S.Since $S \neq \phi$ so fix an $s \in S$.Now $a_s \in N_S$.

If $aD(x) = 0,$ for all $x \in N$, then for all $x_t \in N_S$, where $x \in N, t \in S$, we have $a_s D_*(x_t) = a_s (D(x))_t = (aD(x))_{st} = 0_{st} = 0$, and if $D(x)a = 0,$ for all $x \in N$, then by the same technique we get $D_*(x_t)a_s = 0,$ for all $x_t \in N_S$.Hence N_S is a zero-symmetric left prime near-ring with D_* a derivation on N_S and $a_s \in N_S$ such that $a_s D_*(x_t) = 0 (or\, D_*(x_t)a_s = 0),$ for all $x_t \in N_S$(as $aD(x) = 0$ or ($D(x)a = 0),$ for all $x \in N$).Hence by (**Theorem A),** we get $a_s = 0$ or $D_* = 0$.

If $a_s = 0$ then there exists $u \in S$ such that $ua = 0$.So $0 \neq u \in S \subseteq Z(N)$. As $Z(N)$ contains no proper zero divisors we get $a = 0$.

If $D_* = 0$.Let $x \in N$ be any element, then $x_s \in N_S$.Hence $(D(x))_s = D_*(x_s) = 0$. So that there exists $v \in S$ such that $vD(x) = 0$, and as $Z(N)$ contains no proper zero divisors we get $D(x)=0$.This result is true for all $x\in N$ which means $D=0$ ♦.

Theorem (2):

Let N be a zero-symmetric left centrally prime near-ring of $2-$torsion free in which $Z(N)$ has no proper zero divisors.If D and D' are two centrally zero derivations on N such that $D(a)D'(b)=D'(b)D(a)$, for all $a,b\in N$, then DD' is a derivation on N if and only if $D=0$ or $D'=0$.

Proof:

If $D=0$ or $D'=0$ then $DD'=0$ which is a derivation on N.
Conversely, let DD' be a derivation on N.Since $Z(N)$ has no proper zero divisors so by **Prop. (4),** $S=Z(N)-\{0\}$ is a multiplicative system in N.Then by **Prop. (5),** N_S is a zero-symmetric.Hence N_S is a zero-symmetric left prime near-ring, also by **Prop. (2),** N_S is a $2-$torsion free.Now consider the induced derivations D_* and D'_* on N_S.To show $D_*D'_*$ is a derivation on N_S.Clearly $D_*D'_*$ is a mapping.Now let $a_s, b_t\in N_S$,then

$D_*D'_*(a_s+b_t)=D_*D'_*((at+bs)_{st})=D_*(D'(at+bs))_{st}=$
$(DD'(at+bs))_{st}=(DD'(at)+DD'(bs))_{st}=((DD'(a))t)_{st}+((DD'(b))s)_{st}=$
$(DD'(a))_s t_t+(DD'(b))_t s_s=(DD'(a))_s+(DD'(b))_t=$
$D(D'(a))_s+D(D'(b))_t=D_*(D'(a))_s+D_*(D'(b))_t=D_*D'_*(a_s)+D_*D'_*(b_t)$.

Hence $D_*D'_*$ is an additive mapping.

Also we have

$D_*D'_*(a_s b_t)=D_*D'_*((ab)_{st})=D_*(D'(ab))_{st}=(DD'(ab))_{st}=$
$(aDD'(b)+DD'(a)b)_{st}=(aDD'(b))_{st}+(DD'(a)b)_{st}=$
$a_s(DD'(b))_t+(DD'(a))_s b_t=a_s D_*D'_*(b_t)+(D_*D'_*(a_s))b_t$.

Thus $D_*D'_*$ is a derivation on N_S.Now for all $a_s, b_t\in N_S$ we have $D_*(a_s)D'_*(b_t)=(D(a))_s(D'(b))_t=(D(a)D'(b))_{st}=(D'(b)D(a))_{ts}=$ $(D'(b)D(a))_{ts}=(D'(b))_t(D(a))_s=D'_*(b_t)D(a_s)$.Thus N_S is a 2- torsion

free zero-symmetric left prime near-ring with D_*, D'_* derivations on N_S such that $D_* D'_*$ is a derivation on N_S and $D_*(a_s)D'_*(b_t) = D'_*(b_t)D_*(a_s)$, for all $a_s, b_t \in N_S$.Hence by **(Theorem C),** we get $D_* = 0$ or $D'_* = 0$ If $D_* = 0$, then as $Z(N)$ contains no proper zero divisors we get $D = 0$(as the same argument in
Theorem 1), and if $D'_* = 0$ then $D' = 0$ ♦.

Theorem (3):

Let N be a zero-symmetric left centrally prime near-ring of $2-$torsion free in which $Z(N)$ has no proper zero divisors.If $D : N \to N$ is a centrally zero derivation on N such that $D^2 = 0$ then $D = 0$.

Proof:

Clearly $S = Z(N) - \{0\}$ is a multiplicative system in N.(since $Z(N)$ has no proper zero divisors).Then (from **Prop. (5)** and **Prop.(2)**), N_S is a zero-symmetric left prime near-ring of $2-$torsion free.Now let $D_* : N_S \to N_S$ be the induced derivation on N_S.Then if $a_s \in N_S$ is any element, then by **Prop. (1),** we get
$D_*^2(a_s) = 0$.Thus we get $D_*^2 = 0$.

Hence by (**Theorem D),** we get $D_* = 0$ which gives $D = 0$(as $Z(N)$ contains no
proper zero divisors) ♦.

Section 3: Zero-symmetric left centrally prime near-rings which are abelian

In this section we look for those conditions under which a zero-symmetric left centrally prime near-ring becomes abelian and we give below a sequence of theorems in each a condition is given that makes a zero-symmetric left centrally prime near-ring an abelian near-ring.

Theorem (4):

Let N be a zero-symmetric left centrally prime near-ring in which $Z(N)$ has no proper zero divisors and if D and D' are two nonzero centrally zero derivations on N and $D(a)D'(b) = -D'(a)D(b)$, for all $a, b \in N$ then N is an abelian near-ring.

Proof:

Since $Z(N)$ has no proper zero divisors so by **Prop. (4),** $S = Z(N) - \{0\}$ is a multiplicative system in N for which $[S, N] = \{0\}$, also by **Prop. (5),** N_S is zero- symmetric.Hence N_S is a zero-symmetric left prime near-ring.Now consider the induced derivations D_* and D'_* on N_S.To show $D_* \neq 0$.

Let $D_* = 0$.If $x \in N$ is any element and by fixing an $s \in S$ (since $S \neq \phi$), we get $(D(x))_S = D_*(x_S) = 0$.Hence there exists $t \in S$ such that $tD(x) = 0$.Since $Z(N)$ is without proper zero divisors and $t \neq 0$(because $0 \notin S$) so $D(x) = 0$.The last result is true for all $x \in N$ which means that $D = 0$ which is a contradiction.

Hence $D_* \neq 0$. In a similar way we can get $D'_* \neq 0$.

Now for all $a_u, b_v \in N_S$,where $a, b \in N$ and $u, v \in S$, we have $D_*(a_u)D'_*(b_v) = (D(a))_u(D'(b))_v = (D(a)D'(b))_{uv} = (-D'(a)D(b))_{uv} = (-D'(a))_u)D(b))_v = -(D'(a))_u(D(b))_v = -D'_*(a_u)D_*(b_v)$.Thus N_S is a zero-symmetric left prime near-ring and D_* , D'_* are nonzero derivations on N_S such that $D_*(a_u)D'_*(b_v) = -D'_*(a_u)D_*(b_v)$, for all $a_u, b_v \in N_S$.Hence by (**Theorem B),** we get that N_S is an abelian left near-ring.It remains to show that N is abelian.So let $x, y \in N$, then $x_S, y_S \in N_S$ and $x_S + y_S = y_S + x_S$.Hence $(x+y)_S = (y+x)_S$.That is $(x+y, s) \sim (y+x, s)$ and hence there exists $r \in S$ such that $r((x+y)s - (y+x)s) = 0$ or $rs((x+y) - (y+x)) = 0$.Since $Z(N)$ has no proper zero divisors and $0 \neq rs \in S = Z(N) - \{0\}$) so we get $(x+y) - (y+x) = 0$ or $x + y = y + x$.Hence N is a left abelian near-ring ♦.

Theorem (5):

Let n be a positive integer such that N is an $n-$torsion free zero-symmetric left centrally prime near-ring in which $Z(N)$ has no proper zero divisors and $D: N \to N$ is a centrally zero derivation on N with $D^n(N) \subseteq Z(N)$.

If $D(Z(N)) \neq \{0\}$ then $D^n(Z(N)) \neq \{0\}$ and $(N,+)$ is a left abelian near-ring.

Proof:

Since $Z(N)$ has no proper zero divisors so by **Prop. (4),** $S = Z(N) - \{0\}$ is a multiplicative system in N with $[S,N] = \{0\}$.
Also by **Prop. (2)** and **Prop. (5),** N_S is an $n-$torsion free zero-symmetric left prime near-ring.Now consider the induced derivation $D_* : N_S \to N_S$ on N_S.
Hence by (**Theorem E),** we get $D_*^n(Z(N_S)) \neq \{0\}$ and $(N_S,+)$ is an abelian left near-ring.Let $D^n(Z(N)) = \{0\}$.If $\alpha \in D_*^n(Z(N_S))$ we get $\beta \in Z(N_S)$ such that $\alpha = D_*^n(\beta)$ but then $\beta = \gamma_w$ for $\gamma \in N, w \in S$.To show $\gamma \in Z(N)$.Let $x \in N$, then $x_w \in N_S$.Hence $\beta x_w - x_w\beta = 0$ or $\gamma_w x_w - x_w \gamma_w = 0$ which means that $(\gamma x - x\gamma)_{ww} = 0$.So there exists $\delta \in S$ such that $\delta(\gamma x - x\gamma) = 0$.As $Z(N)$ contains no proper zero divisors we get $\gamma x - x\gamma = 0$ or $\gamma x = x\gamma$.Thus $\gamma \in Z(N)$ and $D^n(\gamma) = 0$.Now $\alpha = D_*^n(\beta) = D_*^n(\gamma_w) = (D^n(\gamma))_w = 0_w = 0$.
So $D_*^n(Z(N_S)) = \{0\}$.This is a contradiction.Thus $D^n(Z(N)) \neq \{0\}$.It remains to show N is an abelian left near-ring, so let $\lambda, \theta \in N$ then $\lambda_S, \theta_S \in N_S$ and N_S being abelian so $\lambda_S + \theta_S = \theta_S + \lambda_S$ or $(\lambda + \theta)_S = (\theta + \lambda)_S$ and hence there exists $\sigma \in S$ such that $\sigma((\lambda + \theta) - (\theta + \lambda)) = 0$ and since $Z(N)$ has no proper zero divisors so $(\lambda + \theta) - (\theta + \lambda) = 0$ or $\lambda + \theta = \theta + \lambda$.Hence $(N,+)$ is an abelian left near-ring ♦.

Theorem (6):

If N is a zero-symmetric left centrally prime near-ring with $Z(N)$ has no proper zero divisors, and it admits a nonzero centrally zero Daif 1-derivation, then $(N,+)$ is abelian.

Proof:

Since $Z(N)$ has no proper zero divisors so by **Prop. (4)**, $S = Z(N) - \{0\}$ is a multiplicative system in N, and from **Prop. (5),** we get N_S is a zero-symmetric left prime near-ring.Let D_* be the induced derivation on N_S.Then from **Prop. (3),** we get that D_* is a Daif 1-derivation.Hence N_S is a zero-symmetric left prime near-ring and D_* is a Daif 1-derivation on N_S.Thus from (**Theorem F),** we get that $(N_S, +)$ is abelian and as $Z(N)$ has no proper zero divisors $(N, +)$ is an abelian group ♦.

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Journal of Kirkuk University –Scientific Studies , vol.2, No.1,2007

حول الحلقات المقتربة اليسارية والاولية مركزيا ذات التناظرالصفري

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الخلاصة

في هذا البحث قدمنا تعريف الحلقات المقتربة اليسارية والاولية مركزياذات التناظرالصفري حيث تمكننا من برهان بعض النتائج الاولية والتي قادتنا الى الحصول على بعض من خواص هذه الحلقات و من ثم الحصول على شروط عديدة والتي يجعل كل واحد منها من الحلقة المقتربة اليسارية والاولية مركزيا ذات ا لتناظرالصفري ابلية.