

Two - mirror resonator as a high resolution length – sensing interferometer

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Abstract

Same basic properties of resonators from a wave – optical and from a ray resonator were investigated. These properties play a pivoted role in determining both the spectral properties of optical resonators and the behavior of rays inside such resonators. In this investigation, the phase was used as a unifying concept. Furthermore the basic expressions for paraxial two mirror resonators were considered a long with a special case where the phase (θ) assumes a rational value, $\theta=2 \pi k \setminus N$. In this case a paraxial two-mirror resonator has, at the same time, a highly degenerate eign frequency spectrum and supports closed periodic orbits that repeat after N round trips. Outside the paraxial limit, these periodic orbits are still useful they allow us to make general statements about the behavior of the phase for non paraxial eign modes. Also extend the expressions for the paraxial two-mirror resonator so that can be used to analyse paraxial resonators with astigmatic eign modes. Also, I suggest a possible application of a degenerate two-mirror resonator as a high- resolution length- sensing interferometer. I discuss its principle of operation and point out some limitations.

Introduction

At the start of the last decade of the 19th century, Gouy (Klaassen *et al.*, 2004, Dingjan *et al.*, 2002) showed that a beam of light that passes through a focus esquires an additional phase factor π . He demonstrated this using a variation on the Fresnel double – mirror experiment, where one of the two mirrors was replaced by a concave mirror. Further study showed that the magnitude of Phase depends on the transverse Structure of the focused beam. For well – behaved, smooth beams without additional structure, this additional phase equals π . Careful analysis has shown that this change in phase does not occur suddenly occurs gradually in a region around the focus. Since at first glance the Phase appears to be a purely wave- optical property, it is surprising to find that it also plays a prominent role in determining the ray properties of resonators. This is a direct consequence of the fact that Gaussian beam propagation rules are the same as the rules that govern paraxial ray optics.

In this search draw together equations and concepts concerning wave optics and ray optics in resonators of various types, using the Phase as a

unifying concept. Most of these results are not new, but are phrased in ways that are useful for the rest of this research. The combination of optics emphasizes aspects of optics that rarely get much attention in textbooks or scientific journals. Our discussion naturally leads to speculation on cases that are not covered by standard theory, most notably non paraxial resonators. In the next section recall the familiar expressions for paraxial two-mirror resonators that can be found in any textbook on laser physics and resonator optics (Siegman, 1986, Milonni & Eberly, 1988 and Verdeyen, 1989), and highlight the parallels between wave and ray optics in such systems. Then, in section 4 I will consider so called degenerate paraxial two-mirror resonators, where the Phase is a rational fraction of 2π . Next, in section 5, we will consider what happens to those periodic orbits, and hence to that special behavior, outside the paraxial limit and I will speculate about the Phase for non paraxial modes. In section 6, consider the Phase in the context of three-mirror resonators, where the resonator eigenmodes are astigmatic. Also give some attention to the possibility of closed periodic orbits in this type of resonators, and briefly speculate about three-mirror resonators outside the paraxial limit. Finally, in section 7 discuss the possibilities of using a degenerate resonator to create a high – resolution optical interferometer.

Paraxial Two-Mirror Resonators

1-Wave– optical perspective

The simplest possible optical resonator is a stable paraxial two-mirror resonator. Its eigen modes are the familiar Hermit-Gaussian or Laguerre-Gaussian modes. For a fundamental Gaussian beam, calculating the phase is straightforward. If we choose $z=0$ at the location of the focus of the beam, and calculate the phase difference relative to this focus find that:

$$\phi(z) = \arctan\left(\frac{z}{z_R}\right) \quad \dots(1)$$

Where z_R is the Raleigh range of the beam (Siegman, 1986). The Phase upon going from a plane at z_0 to a plane at z_1 is then given by

$$\phi_{0,1} = \phi(z_1) - \phi(z_0) = \arctan\left(\frac{z_1}{z_R}\right) - \arctan\left(\frac{z_0}{z_R}\right) \quad \dots(2)$$

The round trip Phase θ for the fundamental Gaussian mode of a resonator can be found by choosing an arbitrary reference plane z_r inside the resonator, and calculating the total phase going from this reference plane to the plane of one of two end mirrors, located at z_2 , then to other end

mirror at z_1 (taking into account the fact that we are now traveling in the opposite direction), and back again to z_r ,

$$\theta = [\phi(z_2) - \phi(z_r)] + [-\phi(z_1) + \phi(z_2)] + [\phi(z_r) - \phi(z_1)] = 2[\phi(z_2) - \phi(z_1)] \quad \dots(2a)$$

Where we have assumed that $z_1 < z_r < z_2$. we see that this result is independent of the choice of reference plane. One useful way to parameterize the geometry of a two- mirror resonator is through the so called "g parameters" $g_{1,2} = 1 - L/R_{1,2}$, where L is the length of the resonator and R_1 and R_2 are the radii of curvature of the two mirrors. We then find, from elementary geometrical optical considerations (Siegman, 1986), that the case $0 \leq g_1 g_2 \leq 1$ corresponds to a so called stable resonator. In a wave – optical context, a resonator is called stable when its eign modes have finite trans- verse dimensions on the mirrors. The $1/e_2$ intensity radii of the spots on the mirrors w_1 and w_2 , as well as the $1/e_2$ – radius of the focus of the beam w_0 (the " waist"), can be calculated using (Milonni & Eberly, 1988).

$$w_1^2 = \frac{\lambda L}{\pi} \sqrt{\frac{g_2}{g_1(1 - g_1 g_2)}} \quad \dots(3)$$

$$w_2^2 = \frac{\lambda L}{\pi} \sqrt{\frac{g_1}{g_2(1 - g_1 g_2)}} \quad \dots(4)$$

and

$$w_0^2 = \frac{\lambda L}{\pi} \sqrt{\frac{g_1 g_2 (1 - g_1 g_2)}{(g_1 + g_2 - 2g_1 g_2)^2}} \quad \dots(5)$$

In Fig.1, plot the stability diagram for two – mirror resonators where we have indicated three familiar configurations.

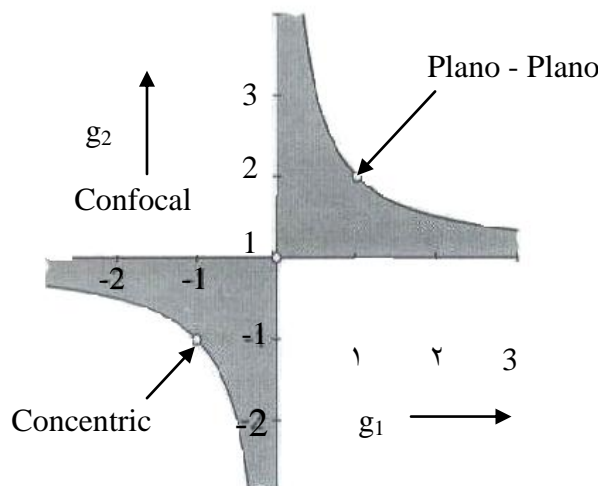


Fig.1: Stability diagram or g_1 - g_2 diagram for two-mirror optical resonators, where $g_{1,2} = 1 - L/R_{1,2}$. The shaded region $0 \leq g_1 g_2 \leq 1$ contains all stable resonator configurations, where, in the paraxial limit, light remains

confined inside a resonator indicated are the Plano- Plano, confocal and concentric resonator configurations (Milonni & Eberly , 1988).

To find the eign modes of a resonator parameterized by g_1 and g_2 , we require the curvature of the wave fronts at the mirrors to match the shape of those mirrors, as in Fig.2.

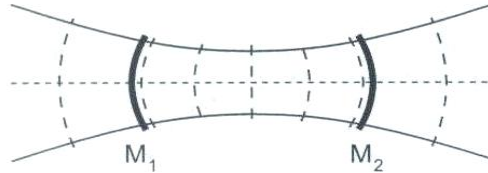


Fig.2: schematic indication of the wave fronts of the eign modes of a two-mirror resonator. At the two-mirrors, the wave fronts must match the curvature of the mirrors.

Then Rayleigh range z_R and the locations of the mirrors z_1 and z_2 are uniquely determined by the resonator length $L=z_2-z_1$ and the radii of curvature of the mirrors R_1 and R_2 . For the Phase, only the ratios z_1/z_R and z_2/z_R are relevant. Expressed in terms of g_1 and g_2 these ratios are:

$$\frac{z_{1,2}}{z_R} = \frac{g_{2,1}(1 - g_{1,2})}{\sqrt{g_1 g_2 (1 - g_1 g_2)}} \quad \dots(6)$$

Substituting Eq.1 into Eq 2a we find, after some algebra that for the round trip Phase.

$$\theta = 2arccos(\pm \sqrt{g_1 g_2}) \quad \dots(7)$$

For a Hermit – Gaussian mode with transverse mode indices m and n , or for a Laguerre – Gaussian mode with transverse mode indices p and L , the round trip Phase is given by

$$\begin{aligned} \theta_{0,n} &= (0 + n + 1)\theta \\ \text{and} \\ \theta_{p3} &= (2P + 3 + 1)\theta \end{aligned} \quad \dots(8)$$

The Eign frequencies of a two – mirror paraxial resonator are then given by

$$\nu_{q,0n} = \frac{c}{2L} \left(q + \frac{\theta_{0n}}{2\pi} \right) = \frac{c}{2L} \left\{ q + (0 + n + 1) \frac{\theta}{2\pi} \right\} \quad \dots(9)$$

The resulting spectrum consists, for every longitudinal mode index q , of combs of equidistant frequencies, where the distance between frequencies is the transverse mode spacing $\Delta\nu_L = c/2L. \theta/2\pi$. These combs

are offset by the longitudinal mode spacing $\Delta\nu_L = c/2L$, as illustrated in Fig.3.

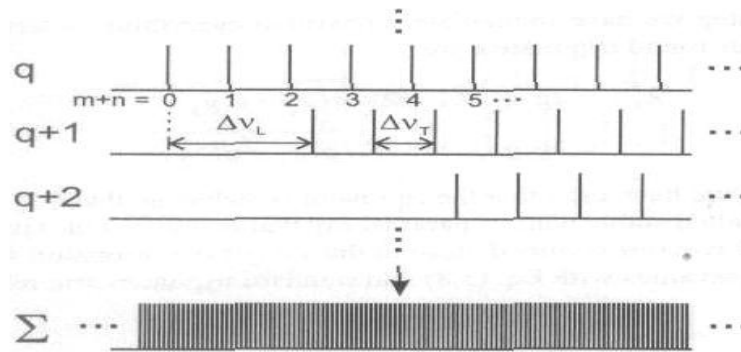


Fig.3: Schematic representation of the structure of the eigen frequency spectrum of paraxial two- mirror resonator. Indicated are, for several values of q, the equidistant frequency combs, with spacing between the peaks of $\Delta\nu_T$. The combs are offset by the longitudinal mode spacing $\Delta\nu_L$. Also indicated is the full spectrum resulting from summing over all possible values for q. (Siegman, 1986).

In this figure, all transverse modes are indicated by bars of the same height. We see that modes that have equal longitudinal mode indices q and equal sums of the transverse mode indices m+n have the same frequency. Therefore, every eigen frequency in the comb corresponding to a single value for q is a (m+n+1) fold degenerate of transverse modes.

Ray-Optical perspective

To describe the ray behaviors of a paraxial two-mirror resonator, we consider a single plane containing the optical axis, and locally describe each ray by a vector $r=(x, x')$. This vector contain the distance of the ray to the axis x and its slope x' in a given reference plane perpendicular to the axis. Next, we calculate the round trip ABCD- matrix, where we choose mirror M_1 as reference plane. Propagation from mirror to mirror, over a distance L, is given by (Kogelnik et al., 1966):

$$M_L = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \quad \dots(10)$$

While the focusing effect of mirrors M_1 and M_2 is described by

$$M_{1,2} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \\ R_{1,2} & \end{pmatrix} \quad \dots(11)$$

The full round trip ABCD- matrix then equal

$$M_{rt} = M_1 M_L M_2 M_L = \begin{pmatrix} -1+2g_2 & 2Lg_2 \\ \frac{2g_1g_2-g_1-g_2}{L} & 4g_2g_2-2g_2-1 \end{pmatrix} \quad \dots(12)$$

Where in the second step we have immediately rewritten everything in terms of g_1 and g_2 . The eigen values for this round trip matrix are:

$$\lambda_{1,2} = 2g_1g_2 - 1 \pm 2\sqrt{g_1^2g_2^2 - g_1g_2} = 2g_1g_2 - 1 \pm 2i\sqrt{g_1g_2 - g_1^2g_2^2} \quad \dots(13)$$

Where, in the last step, we have used that the resonator is stable, so that ($0 \leq g_1g_2 \leq 1$). In ray optics, a resonator is called stable when a paraxial ray that is injected into the resonator does not escape, but instead remains confined close to the axis of the resonator. Combining these expressions for the eigen values with eq. (7) and standard trigonometric relations, we find

$$\lambda_{1,2} = \exp(\pm i\theta) \quad \dots(14)$$

Degenerate paraxial two – mirror resonators.

The discussion in the previous section is valid for all possible values of the Phase ($0 \leq \theta \leq 2\pi$). However, when the Phase is a rational number times 2π , that is $\theta = 2\pi K/N$, where K and N do not share common factors, these equations have unexpected consequences. Then, Eq. (9) can be rewritten to:

$$v_{q,mn} = \frac{c}{2L} \left(q + (m+n+1) \frac{K}{N} \right) = \frac{c}{2LN} \{ Nq + K(m+n+1) \} \quad \dots(15)$$

From this we see that raising the sum of the transverse mode indices $m+n$ by N, while at the same time lowering the longitudinal mode index q by K, will leave the resonance frequency unaltered, irrespective of the choice of q, m or n . Therefore, a part from the mode degeneracy within "families" of transverse modes mentioned in section 2, the resonator now also shows degeneracy between different transverse mode families, separated by K in longitudinal mode index and N in total transverse mode index. The entire eigen frequency spectrum collapses into these strongly degenerate clumps, spaced at $\Delta v = 1/N \Delta v_L = c/2LN$, Fig.4. Thus, a single free spectral range will contain N of these degenerate clumps of modes, a fact that was first observed as early as 1964 (Herriott *et al.*, 1964).

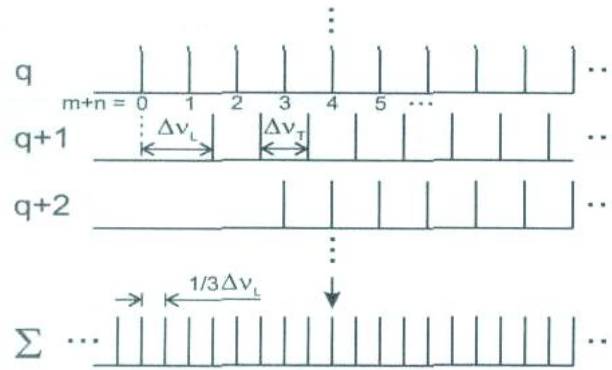


Fig.4: Schematic representation of the structure of the eigen frequency spectrum of a typical degenerate paraxial two-mirror resonator, with $\theta= 2\pi \cdot 2/3$. Indicated for three values of q , the equidistant frequency combs, with spacing between the peaks of $\Delta v_T=2/3\Delta v_L$. The combs are offset by the longitudinal mode spacing Δv_L . Also indicated is the full eigen frequency spectrum obtained from summing over all possible q . (Herriott *et al.*, 1964).

The eigen values of the round trip matrix of a resonator for which $\theta= 2\pi K/N$ follow from eq. (14):

$$\lambda_{1,2} = \exp(\pm i2\pi(K/N)) \quad \dots(16)$$

We see that for degenerate resonators, we can distinguish two basic classes of periodic orbits. The first class consists of generic orbit, where the ray never strikes a mirror at normal incidence, such as in Fig. 5b and 5e. The second class consist of reciprocating orbits, where the orbit strikes a mirror at normal incidence, reverse direction and re-traces itself, as in Fig. 5a,c,d and f.

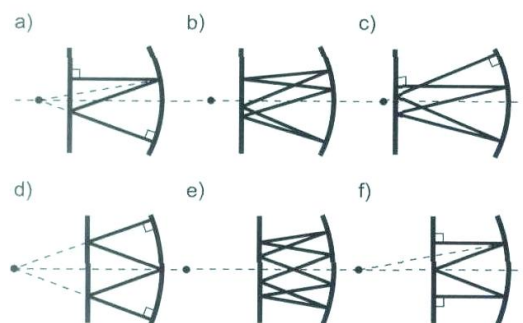


Fig.5: Typical closed periodic orbits in degenerate resonators. (a) $K/N =1/3$, reciprocating path ("Z-shaped"),(b) $K/N=1/3$, generic path, (c) $K/N=2/5$, reciprocating path, (d) $K/N=1/4$, reciprocating path ("W-shaped"), (e) $K/N=1/4$, generic path, (f) $K/N=1/4$, reciprocating path ("M-shaped").(Herriott *et al.*, 1964)

Folded three-mirror resonators

A more complicated optical resonator than the two-mirror resonators considered so far is a folded three- mirror resonator. Our interest in this type of resonator stems from the fact that. In Fig.6 we draw a simple folded three-mirror resonator.

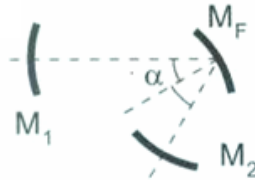


Fig.6: Schematic drawing of a three-mirror resonator. M_F is the curved folding mirror; M_1 and M_2 are the resonator end mirrors. The resonator is folded through 2α , so that the angle of incidence of the axis on the folding mirror M_F is α .

It is essential that the folding mirror M_F is a curved mirror. If not, the resonator can be trivially transformed back into a two-mirror resonator. Because of the non- normal incidence on the folding mirrors, the effective radius of curvature of that mirror, and hence the focal power, will be different for different planes containing the axis. Here, it is sufficient to consider the behavior in two perpendicular planes, both containing the optical axis, the so called principal planes.

The first of these is the plane defined by the two segments of the optical axis. This is customarily called the tangential plane, and the effective radius of curvature of the folding mirror in this plane is (Dingjan et al., 2002):

$$R_T = R \cdot \cos\alpha \quad \dots(17)$$

When the folding angle 2α increases, R_T will decrease and hence the focusing effect of the folding mirror in the tangential plane will increase.

The second plane is the plane perpendicular to the tangential plane and containing the optical axis. Because the optical axis is folded, this plane is, real space, folded as well. Customarily, it is called the sagittal plane. The effective radius of curvature of the folding mirror in this plane is (Dingjan et al., 2002):

$$R_S = R / \cos\alpha \quad \dots(18)$$

So that, for increasing folding angle 2α , R_S will increase and hence the focal power will decrease.

As a result, the total focal power in a single round trip through the resonator depends on whether one considers the tangential or the sagittal plane. A direct consequence of this is that the eign modes of a paraxial

three-mirror resonator are astigmatic Hermite- Gaussian modes, with generally an elliptical cross-section. Also, instead of a single phase θ to describe the structure of. The eign frequencies of the astigmatic Hermit – Gaussian eign modes of a folded three-mirror resonator are given by Siegman (1986), Hanna (1969).

$$v_{q,mn} = \frac{c}{2L} \left(q + (m + \frac{1}{2}) \frac{\theta_T}{2\pi} + (n + \frac{1}{2}) \frac{\theta_S}{2\pi} \right) \quad \dots(19)$$

Where θ_T and θ_S are the phases for the tangential and the sagittal plane, respectively. These phases depend on the folding angle 2α of the resonator. Using the appropriate effective radii of curvature of Eqs(17and 18) for the folding mirror. The eign values of these round trip matrices will then be $\exp(\pm i\theta_T)$ and $\exp(\pm i\theta_S)$.

In Fig.(7) plot θ_T and θ_S versus half folding angle α , for a resonator with three- mirrors with a radius of curvature $R=1m$. One arm of the resonator has a length $L_1=10cm$, while the other arm has a length $L_2=20cm$.

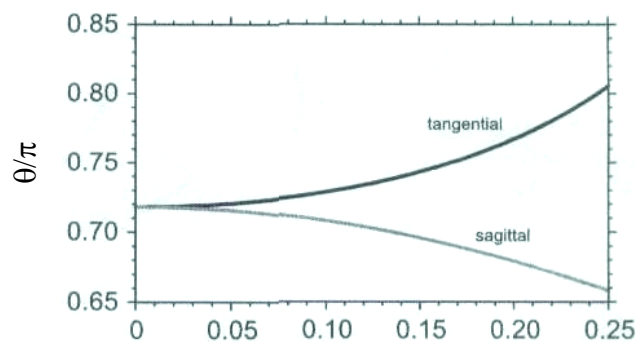


Fig.7: Tangential and sagittal phases θ_T and θ_S versus half folding angle α , for a folded three-mirror resonator consisting of three-mirrors with radius of curvature $R=1m$. The length of one arm of the resonator $L_1=10cm$, while the length of the other arm $L_2=20cm$ (Siegman, 1986).

I can re-order the terms in eq. (19) to get the more practical form.

$$v_{q,mn} = \frac{c}{2L} \left(q + (m+n+1) \frac{\theta_T + \theta_S}{4\pi} + (m-n) \frac{\theta_T - \theta_S}{4\pi} \right) = \frac{c}{2L} \left(q + (m+n+1) \frac{\theta}{2\pi} + \frac{(m-n)}{2} \frac{\delta}{2\pi} \right) \quad \dots(20)$$

Where we have introduced the average phase $\theta = (\theta_T + \theta_S) / 2$ and the phase difference $\delta = (\theta_T - \theta_S)$. Using this equation we see that the eign frequency spectrum of a paraxial folded three- mirror optical resonator has a more interesting structure than that of a two-mirror optical resonator. For every longitudinal mode index q there is a "super-family", an equidistant

series of mode families with constant total transverse mode index $m+n$. The centers of each family are spaced at $\Delta\nu_T = \theta/2\pi \cdot c/2L$, similar to the transverse mode spacing $\Delta\nu_T$ for a two-mirror resonator. From eq.(20) we see, after noting that for a family of modes $m+n$ is a constant so that the difference $m-n$ can only change in steps of 2, that the distance between the different members of a single family is equal to $\Delta\nu_M = \delta/2\pi \cdot c/2L$. (Siegman, 1986).

Degenerate resonator as high-resolution interferometer

As we saw in section 4, a degenerate two-mirror optical resonator has an eigen frequency spectrum consisting of evenly spaced resonance, with an inter-resonance distance of $\Delta\nu=1/N \cdot c/2L$. The best known example of such a degenerate resonator is a confocal Fabry-Perot resonator, with $N=2$, as already mentioned on eq.15 Here, I will consider resonators with much larger N . We may consider using such a degenerate resonator as a high- resolution interferometer for sensing changes in length, as changing the length of the resonator will shift its resonance frequencies. If we inject monochromatic light (He-Ne Laser) at a fixed frequency ν_0 into a resonator, and monitor the amount of transmitted light as the length of resonator changes, I will see maximum transmission when a resonator eigen frequency coincides with the frequency of the injected light. To first approximation, the eigen frequencies of a resonator change linearly with the length of the resonator, and changing the length of the resonator by $\lambda/2$, where λ is the wave length of the He-Ne laser, will shift all frequencies by $\Delta\nu_L = c/2L$, a single free spectral range.

For a degenerate resonator where the distance between resonances is $1/N \cdot c/2L$, that is a factor of $1/N$ smaller than for non degenerate mode-matched interferometer, the change in length of the resonator required to get from one resonance to the next is also a factor $1/N$ smaller, $\delta L = 1/N \cdot \lambda/2$.

In reality, conventional, non degenerate resonators also offer a resolution that is higher than the fundamental mode spacing $\lambda/2$. Because the intensity reflectivity of the mirrors $R_{1,2} < 1$, every resonance has a finite width $\delta\nu$ (FWHM). The ratio of the free spectral range to this line width is called the finesse F , and is, for $R_{1,2}$ close to 1, given by

$$F = \frac{c}{2L} \frac{1}{\delta\nu} = \frac{2\pi}{1-R} \quad \dots(21)$$

With

$$R = R_1 \cdot R_2.$$

There are several limitations to using a degenerate optical resonator as a high- resolution interferometer. The first is that a resonator is only

perfectly degenerate at a single resonator length $L_{K/N}$. Changing this length, even by tiny fractions of $\lambda/2$, will destroy this perfect degeneracy.

A second limitation is that it is not possible to increase N at will. To get a nicely developed degenerate spectrum, with inter-resonance spacing $1/N \cdot c/2L$, one wishes to excite more than N families of modes, each with constant $(m+n)$.

A final limitation on a high-resolution interferometer, one that is shared between both conventional and degenerate resonator types, is the line width of individual resonances. Therefore, a resolution that is many orders of magnitude smaller than this minimum line width cannot be reached. However, using present day technology, a finesse F as high as 10^5 or 10^6 is possible allowing, in principle, an inter-resonance spacing as small as $\lambda/10^6 = 0.5$ pm. (K.An et al., 1997, Katherine Kerner, 2000).

Conclusion

1. I have seen in this research that consideration of both wave and ray properties of optical resonators in combination can help improve the understanding of both. There exist close ties between the structures of the eigen frequencies spectrum of a resonator.
2. The behavior of rays in the resonator's a result, a convenient way of calculating.
3. The phase of a resonator, $\theta=2\pi k \setminus N$, that resonator has at the same time a highly degenerate eigen frequency spectrum, and supports periodic orbits.
4. At least some of these periodic orbits survive outside the paraxial limit, but, where in the paraxial limits all periodic orbits for a given K/N occur at the same resonator length $L_{K/N, \text{parax}}$, non paraxial periodic K/N -orbits are only possible at resonator lengths that depend strongly on the exact shape of the orbit.
5. The behavior of these periodic orbits outside the paraxial limit, we may conclude that the round trip phase for none paraxial eigen modes will be larger than the paraxial phase.
6. A full calculation of non paraxial eigen modes has to be performed.
7. A folded three-mirror optical resonator has properties that resemble those of a two-mirror resonator. A crucial difference is that eigen modes are astigmatic, so that now phase's θ_T and θ_S are needed to fully describe both the structure of the eigen frequency spectrum and general ray behavior inside such a resonator.
8. Degenerate eigen frequency spectra and periodic orbits are still possible, but now with the stricter requirement that both θ_T and θ_S are rational fractions of 2π .

9. We saw that a degenerate two-mirror resonator may be used as a high-resolution interferometer for length –sensing purposes, where the benefit over a more conventional standing wave interferometer lies in the extended working rang.

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مرنان ذي مراتين كمدخال حساس ذي قابلية تفريقية عالية

سوران ذوالنون ياسين

كلية الزراعة _ جامعة صلاح الدين / اربيل

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الخلاصة

خلال هذه الدراسة، حققت بعض خصائص للمرنان البصرى - الموجي و المرنان الحزمي. وتلعب هذه الخصائص دوراً فعالاً لتحديد صفات الطيفية للمرنان البصرى و سلوك الحزم داخل المرنان . استخدم الطور كفكرة توحيدية. اضافة الى ذلك لوحظ الصيغ الاساسية للمحاور المرنان المزدوج كحالة خاصة حيث أن الطور يمكن افتراضه بأن له قيمة صحيحة $(\theta = 2\pi k/N)$. في هذه الحالة المحاور المرنان المزدوج له في نفس الوقت، انحلال عالي لتردد ذاتي للطيف ويزود مدارات مغلقة دورية حيث يتكرر بعد N من دورات المعادة.

هذه المدارات دورية تكون مفيدة خارج نطاق هذا المحور لتسمح لنا جعل صيغة عامة حول سلوك للطور غير محموري لانماط الذاتية - بالاضافة قمت بتمديد صيغ المحاور المرنان المزدوج حيث يمكن استخدام لتحليل محاور مرنان مع عيوب انماط ذاتية. اقترحت تطبيقاً ممكناً لانحلال المرنان المزدوج كتفريق عالي لطول المدخال الحساس ومبادئه الاساسية للعملية وملاحظة تقيدات .