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Optimal Transformation Technique To Solve Multi-Objective Linear Programming Problem (MOLPP)

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Abstract

In this paper ,we suggested a new technique by using optimal average(O_{AV}) for function. and an algorithm is suggested for it is solution .The MOLPP criteria of Chandra Sen and Sulaiman & Sadiq(Sen,Chandra,1983 ; Sulaiman & Sadiq, 2006, respectively) ; has been modified in this paper. The computer application of algorithm also has been demonstrated by a flow-cart and solving a numerical examples .The numerical results in(Table3) indicate that the new technique in general is promising.

Introduction

In (1983),Chandra Sen (Sen,1983) defined the multi-objective linear programming problem, and suggested an approach to construct that the multi-objective function under the limitation that the optimum value of individual problem is greater than zero. In (1992), Sulaiman and Mohammad (Sulaiman & Mahammad, 1992) studied the multi-objective fractional complimentary program. In(1993),Abdil-Kadir and Sulaiman (Abdul Kadir & Sulaiman, 1993) studied the multi-objective fractional programming problem .In (2006),Sulaiman and Sadiq (Sulaiman & Sadiq, 2006) studied the multi-objective function by solving the multi-objective programming problem ,using mean and mean value ;and they did try optimal solution and comparison results between Chandra Sen approach (Sen, 1983) and modified approach (Sulaiman & Sadiq, 2006). In order to extended this work we have defined a multi-objective programming problem linear and investigated the algorithm to solve linear programming problem for multi-objective functions (Sen, 1983).Irrespective of the number objectives with less computational burden and suggest a new technique by using optimal average (O_{AV}) of objective functions; to generate the best optima solution. The computer application of our algorithm also has been discussed by solving a numerical examples. Finally we have been shown results and comparison they between the new technique and Chandra Sen approach (Sen, 1983) & Sulaiman approach (Sulaiman & Sadiq, 2006).

Mathimatical form of the multi-objective programming problem

A multi-objective linear programming (MOLPP) is introduced by Chandra Sen (Sen, Chandra, 1983) and suggested an approach (C_A) to construct the multi-objective function under the limitation that the optimal value of individual problem is greater than zero (Abdul Kadir & Sulaiman, 1993). He has not considered the situation when the optimum value of some of individual objective function functions may be negative or zero also (Sulaiman & Sadiq, 2006).The mathematical form of this type of problem is given as follows:

$$\left.\begin{aligned} &\text{Max } z_1 = c_1^t.x + a_1 \\ &\text{Max } z_2 = c_2^t.x + a_2 \\ &\text{Max } z_r = c_r^t.x + a_r \\ &\text{Min } z_{r+1} = c^t{}_{r+1}.x + a_{r+1} \end{aligned}\right\} \quad \ldots(2.1)$$

.
.
.

$$\text{Min } z_s = c^t{}_s.x + a_s$$

Subject to constraints:

$$A.X = B \quad \ldots(2.2)$$

$$X \geq 0 \quad \ldots(2.3)$$

Where r is the number of objective functions to be maximized, is the number of objective functions to be max & minimized, X is an n-dimensional vector of decision variables, C is n-dimensional vector of constants, B is m dimensional vector of constants, (s-r) is the number of objective functions that is to be minimized, A is a (mxn) matrix of coefficients. All vectors are assumed to be column vectors unless transposed, a_i(i=1,2,…,s) are scalar constants, $C^t.X + a_i$; i=1,2,3….,s, are linear factors for all feasible solutions (Abdul Kadir & Sulaiman,1993).

If $a_i = 0$; $\forall i = 1, 2, \ldots, s$, then the mathematical form Become:

$$\text{Max } z_1 = c_1^t.X$$
$$\text{Max } z_2 = c_2^t.x$$
$$\text{Max } z_r = c_r^t.x$$
$$\text{Min } z_{r+1} = c^t{}_{r+1}.x$$

.
.

$$\text{Min } z_s = c^t{}_s.x$$

Subject to constraints: - $A.X = B$ … (2.2)

$X \geq 0$ … (2.3)

Formulation of multiobjective functions

The same approach taken by sulaiman and Gulnar (Sulaiman & Sadiq, 2006) for multiobjective functions is folled here to for emulate the constrained objective functions given in equation (2.1).suppose we obtained a single value corresponding to each of objective functions of it being optimized individually subject to constraints (2.2) and (2.3) as follows:

Max $z_1 = \Phi_1$

Max $z_2 = \Phi_2$

Max $z_r = \Phi_r$

Min $z_{r+1} = \Phi_{r+1}$

Min $z_s = \Phi_s$

Where $\Phi_i; i=1,2,\ldots,s$ the decision variable may not necessarily be common to all optimal solutions in the presence of conflicts among objectives (Sulaiman & Sadiq, 2006). But the common set of decision variable between object functions are necessary in order to select the best compromise solution (Azapagic, 1999). We can determine the common set of decision variable from the following combined objective function (Sen, 1983;AbdulKadir&Sulaiman,1993 and Sulaiman& Sadiq,2006).

Which formulate the MOLPP given in (2.1) as:

$$\text{Max } Z = \sum_{k=1}^{r} Zk / |\Phi k| - \sum_{k=r+1}^{s} zk / |\Phi k| \quad \ldots(2.5)$$

For all $0 \neq Z_k$;k=1,2,…,s.

Subject to the same constraints (2.2), (2.3); and the optimum value of functions $\Phi_k \in R\backslash\{0\}$; where R is the set of real numbers. Now we can solve this MOLPP by Chardra Sen approach (C_A) (Sen, 1983; Abdul Kadir & Sulaiman, 1993 and Sulaiman & Sadiq, 2006).

Solving the MOLPP by modified approach (M_A)

We formulate the combined objective function as follows to determine the common set of decision variables ,to solving the MOLPP by modified approach (using mean and median value)(Sulaiman & Mohammad, 1992).

$$\text{Max.}Z = \sum_{i=1}^{r} Z_i/\text{mean}(AA_i) - \sum_{i=r+1}^{s} Z_i/\text{mean}(AL_i) \quad \ldots(2\text{-}6)$$

Subject to the same constraints (2-2),(2-3);

Where $AA_i = |\Phi_i|$, for all i=1,2,…r;

$AL_i = |\Phi_i|$, for all i= r+1,r+2 …s

$$\text{Max.}Z=\sum_{i=1}^{r} Z_i/\text{median}\,(AA_i)-\sum_{i=r+1}^{s} Z_i/\text{median}\,(AL_i) \qquad \ldots(2\text{-}7)$$

Subject to the same constraints (2-2), (2-3); where both of AA_i & AL_i the same values of (2-6) respectively.

Solving the MOLPP by using the Optimal Average (O_{AV}):-

O_{AV}:-

Before solving MOLPP, and preface an algorithm to it, we will need to define some definitions:-

Definitions (1):-

let $m_1=\min\{AA_i\}$,where $AA_i=|\Phi_i|$,and Φ_I is the maximum value of Z_i ,for all i=1,2,…r.

Definitions (2):-

Let m2 =min{ALi},where ALi=$|\Phi i|$, and ΦI is the minimum value of Zi ,for all i=r+1,r+2,…s.

Definitions (3):-

We denote the Optimal Average by OAV ,and define it as:-
OAV=(m1+m2)/2 ;where mj defined by Definition(j) ,for all j=1,2 respectively

Algorithm

The following algorithm is to obtain the optimal solution for the multiobjective linear programming problem defined previous can be summarized as follows:-

Step1: Find the value of each of individual objective functions which is to be maximized or minimized.

Step2: slove the first objective problem by simplex method.

Step3: check the feasibility of the solution in step2. if it is feasible then go to step 4, otherwise, use dual simplex methods to remove infeasibility.

Step4: assign a name to the optimum value of the first objective function Z_1 say ϕ Ai

Step5: repeat the step 2; i=1,2,3,4 for the k^{th} objective problem, $\forall$ k=2, 3,---s.

Step6: Select $m_1=\min\{\phi A_i\}, \forall i= i_1,_2\text{---}r, M_2=\min\{\phi A_i\}, \forall i=r+1,r+2,\text{---}s$

Calculate $O_{AV}=\frac{1}{2}(m_1+m_2)$

Step7: Optimize the combined objective function order the same constrains (2.2),(2-3) as :

$$\text{Max.Z}= (\sum_{i=1}^{r} \max.Zi\ .\ -\ \sum_{i=r+1}^{s} \min.Zi)/O_{AV} \qquad \ldots(2.8)$$

By repeating the step i , i=2,3,4.

Program Notation :

The following notations, which are used in computer program are defined as follows:

ϕ A_i= The value of objective function which is to be maximized.

ϕ L_i= The value of objective function which is to be minimized.

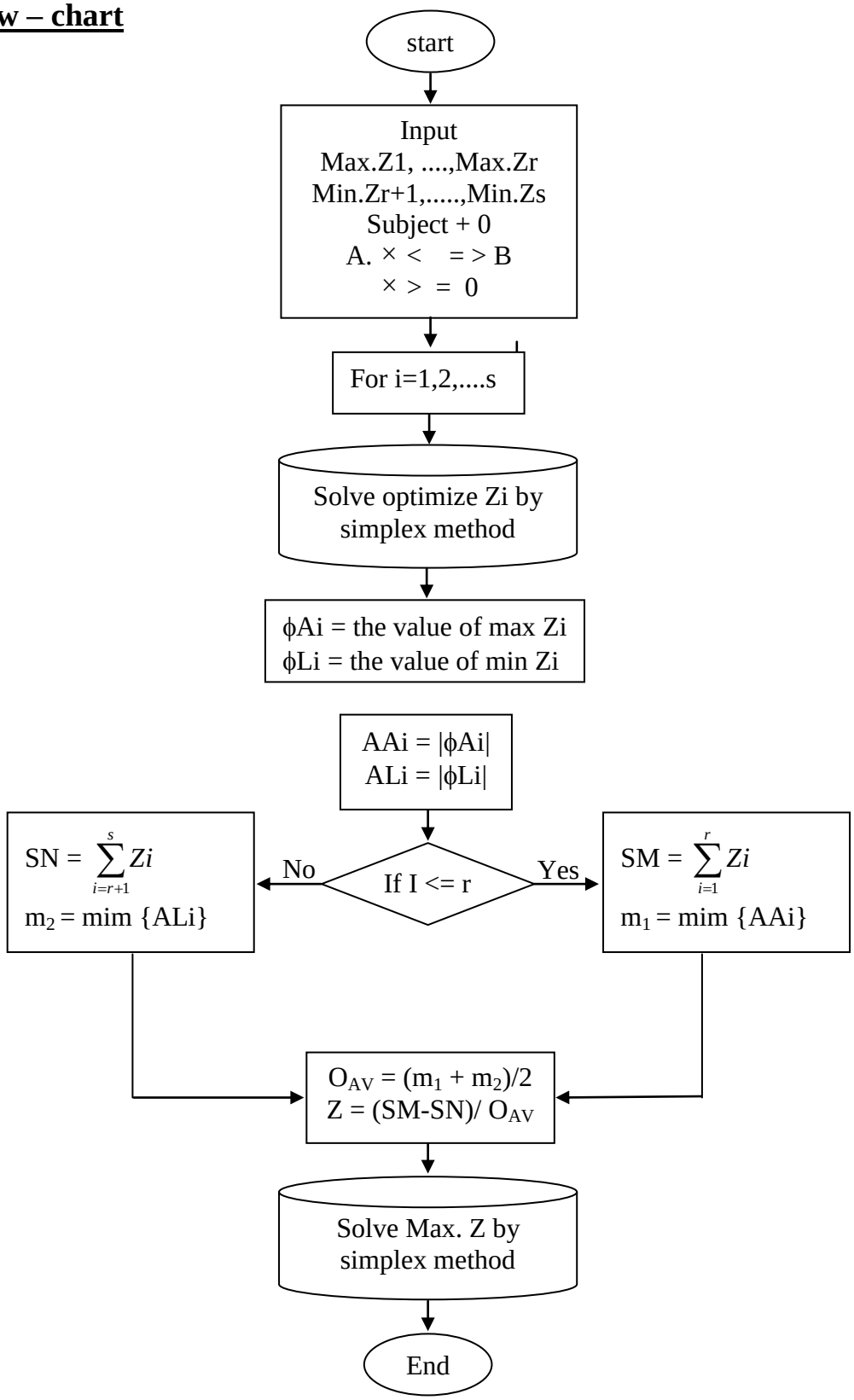
$AA_i=|\phi Ai|$; $\forall_i$= 1,2,.....r

$AL_i=|\phi Li|$; $\forall_i$= r+1,r+2.....s

$SM=\sum_{i=1}^{r} Zi$; $SN=\sum_{i=r+1}^{s} Zi$; $m_1=\min\{AAi\}$, $m_2=\min\{ALi\}$

$O_{AV}=\frac{1}{2}(m_1+m_2)$; Max.Z=(SM– SN)/ O_{AV}

Flow – chart


start
Input
Max.Z1, ...,Max.Zr
Min.Zr+1,.....,Min.Zs
Subject + 0
A. × < = > B
× > = 0
For i=1,2,....s
Solve optimize Zi by simplex method
ϕAi = the value of max Zi
ϕLi = the value of min Zi
AAi = |ϕAi|
ALi = |ϕLi|
If I <= r
No
Yes
SN = $\sum_{i=r+1}^{s} Zi$
m2 = mim {ALi}
SM = $\sum_{i=1}^{r} Zi$
m1 = mim {AAi}
OAV = (m1 + m2)/2
Z = (SM-SN)/ OAV
Solve Max. Z by simplex method
End

Numerical Examples

Ex.(1):

$\text{Max.}Z_1=X_1+2X_2$

$\text{Max.}Z_2=X_1$

$\text{Min.}Z_3= -2X_1-3X_2$

$\text{Min.}Z_4=-X_2$

Subject to =-

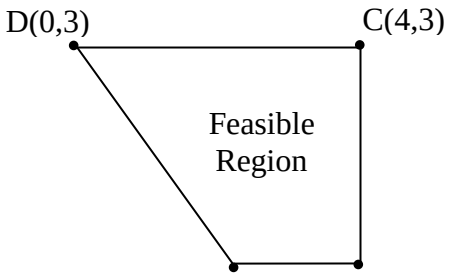
$6X_1+8X_1 \leq 48$

$X_1+ X_2 \geq 3$

$X_1 \leq 4$

$X_2 \leq 3$

$X_1, X_2 \geq 0$



Fig(1)

Solution:

After finding the value of each of individual objective functions by simplex method the results as below (in table 1):

using (2-8) for solve the ex.(1) we get :

$\text{Max.}Z= 1.14285X_1 + 1.71428X_2$

or

$$\left.\begin{aligned} \text{Max.}Z &= 1.14285\,X_1 + 1.71428X_2 \\ &\text{Subject to given constraints:} \\ 6X_1 + 8X_2 &\leq 48 \\ X_1 + X_2 &\geq 3 \\ X_1 &\leq 4 \\ X_2 &\leq 3 \\ X_1, X_2 &\geq 0 \end{aligned}\right\} \quad \ldots(2\text{-}10)$$

Solving (2-10) to obtained optimal solution as:-

$\text{Max.}Z= 9.714424, \quad X_1=4, X_2=3$

Note:

Solve (2-10) by:

1-Chandra sen approach, we get:

$\text{Max.}Z= 3.39996$, $X1=4, X_2=3$

2-modified approach

2.1: using median, we get that $\text{Max.}Z= 3.39999$, $X1=4, X_2=3$

2.2: using median, we get that $\text{Max.}Z= 3.39999$, $X1=4, X_2=3$

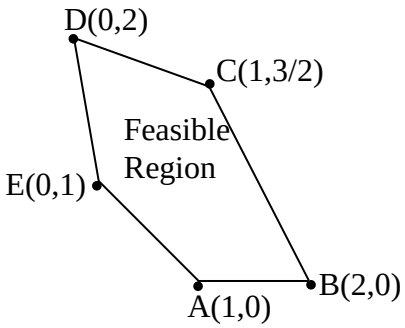
Ex. (2)

$\text{Max.}Z_1= 5 + 2X_1 + X_2$

$\text{Max.}Z_2= 7 + 3X_1 + X_2$

$\text{Max.}Z_3= 6 + 2X_1 + 2X_2$

$\text{Min.}Z_4 = 3 + 3X_1 + X_2$
$\text{Min.}Z_5 = 8 + 3X_1 + 2X_2$
$\text{Min.}Z_6 = 2 + X1 + 3X_2$
s.to:-
$X_1 + X_2 \geq 1$
$3X_1 + 2X_2 \leq 6$
$2X_1 + 4X2 \leq 8$
$X_1 , X_2 \geq 0$



Fig(2)

Solution:
After finding the value of each of individual objective functions by simplex method the results as below:
Using (2-8) for solve the ex. (2) we get:-
$\text{Max.}Z = 0.83333 - 0.33333X_2$
Or
$\text{Max.}Z = 0.83333 - 0.33333\ X_2$
Subject to given constraints:

$$\left.\begin{aligned} 6X_1 + 8X_2 &\geq 1 \\ 3X_1 + 2X_2 &\leq 6 \\ 2X_1 + 4X_2 &\leq 8 \\ X_1 , X_2 &\geq 0 \end{aligned}\right\} \quad \ldots(2\text{-}11)$$

Solving (2-12) by simplex method we get:-
$\text{Max.}Z = 0.83333$ $X_1=1, X_2=0$ or $X_{1=2,}\ X_2=0$
Note:
Solve (2-12) by:
1- Chandra Sen approach, we get that:
$\text{Max.}Z = 1.32574$, $X1=1, X_2=0$
2- Modified approach
2.1: using median, we get that $\text{Max.}Z = -1.25668$, $X1=1, X_2=0$
2.2: using median, we get that $\text{Max.}Z = -2.72728$, $X1=1, X_2=0$

Table(1): results of example(1)

I	Z_i	X_i	ϕ_i	AA_i	AL_i	m_1	m_2	$O_{AV}=\frac{1}{2}(m_1+m_2)$
1	10	(4,3)	10	10		4		3.5
2	4	(4,0),(4,3)	4	4				
3	$^-17$	(4,3)	$^-17$		17	3		
4	$^-3$	(4,3),(0,3)	$^-3$		3			

Table (2): results of example (2)

I	Z_i	X_i	ϕ_i	AA_i	AL_i	m_1	m_2	$O_{AV}=\frac{1}{2}(m_1+m_2)$
1	9	(2,0)	9	9		9		$\frac{1}{2}(9+3)=(\frac{12}{2})=6$
2	13	(2,0)	13	13				
3	11	$(1,\frac{3}{2})$	11	11				
4	4	(0,1)	4		4	3		
5	10	(0,1)	10		10			
6	3	(0,1)	3		3			

Table (3): compare between results obtained by :(C_A),(M_A) & (O_{AV}) approach.

Examples	Chandra Sen approach	Modified approach		Optimal technique approach using O_{AV}
		Using mean	Using median	
Example(1)	Max.Z=3.39996 X_1=4 , X_2=3	Max.Z=3.39999 X_1=4 , X_2=3	Max.Z=3.39999 X_1=4 , X_2=3	Max.Z=9.71424 X_1=4 , X_2=3
Example(2)	Max.Z=-1.32574 X_1=1 , X_2=0	Max.Z=-1.25668 X_1=1 , X_2=0	Max.Z= - 2.72728 X_1=1 , X_2=0	Max.Z=0.83333 X_1=1 , X_2=0 Or X_1=2 , X_2=0

In the table 3 it is clear; the results in optimal approach was better than the results by other approaches.

Conclusion

1- Solving the multi objective programming problem by modified approach takes more consumer time than our optical technique as indicate from their flow – charts and algorithms; since modification approach compute $\sum_{i=1}^{r} \max . Z_i$ / mean (median)- $\sum_{i=r+1}^{s} \min . Z_i$ / mean (median),which more consumer time the member of objective functions be increasing .

2- The results higher by our optimal technique, then by modification approach, even there is only tow objective functions one is to be maximized and the other is minimized.

3- Since the results by modification is better and more optimal than the result by Chandra Sen (Sulaiman & Sadiq, 2006).Hence the result by our optimal technique is more better than the result by Chandra Sen. As indicated in table (3).

4- For all cases the introduced objective function Z is to be maximized.

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تقنية تحويلية المثلى لحل مسألة البرمجة الخطية لمتعددة الأهداف

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الخلاصة

في هذا البحث حاولنا اقتراح تقنية تحولية جديدة لتحويل مسائل البرمجة الخطية لمتعددة الأهداف إلى المسالة البرمجة الخطية لهدف واحد؛ باستعمال المعدل المناسب. كذالك اقترحنا الخوارزمية المناسبة للحل من منطلق بحثي كاندرا سين و سليمان–صادق على التوالي المعدل في هذا البحث.

(Sen, Chandra, 1983; Sulaiman & Sadiq, 2006) مع تطبيق بعض الأمثلة العددية لهذه الخوارزمية المسندة والمدعومة بفلوكارت على الحاسوب. والنتائج في (جدول٣) تبين بوضوح أن هذه الطريقة الأكثر ملائم.