

An Application of Lagrange Multiplier for integer Linear Programming in production- Transportation with Flexible Transportation Cost

Abdul Rahim K.Rahi and Suzan S. Haydar
College of Administration and Economic-University of Sulamani,

Accepted: 2008/4/8 , Received: 2007/6/20

Abstract

The Transportation problem is a classic Operations Research problem where the objective is to determine the schedule for transporting goods from source to destination in a way that minimizes the shipping cost while satisfying supply and demand constraints. Although it can be solving as a linear programming problem. Linear programming makes use of the simplex method, an algorithm invented to solve a linear program by progressing from one extreme point of the feasible polyhedron to an adjacent one. The algorithm contains tactics like pricing and pivoting. For a transportation problem, a simplified version of the regular simplex method can be used, known as the transportation simplex method.

In this paper will discuss the functionality of both of these algorithms, and compared their optimized values with non-linear method called the Lagrange Multiplier Method. Lagrange Multiplier is an algorithm that uses different mechanisms to choose the best optimal solutions. This method based on transforming the linear structure transportation problem into the nonlinear structure and solved it directly, by the techniques. The objective of the study was to find out how these algorithms behave in terms of accuracy and speed when a large-scale problem is being solved.

Transportation Problem

The transportation problem is classic Operation Research problem. Therefore; arises frequently in planning for the distribution of goods and services from several supply locations (Sources) to several demand locations (Destinations) in a way that minimize the shipping cost, while satisfying supply and demand constraints. Typically, the quantity of goods available at each supply location is limited, and the quantity of goods needed at each of several demand locations is known. Therefore; a typical Transportation Problem has the following elements:

1. Source(s)
2. Destination(s)
3. Weighted edge(s)

The objective of a Transportation Problem is to determine the Number

of units to be transported from source (i) to destination (j) so that the total transportation cost is minimum (Phillips et al.,1976 ; Gupta & Hira, 2002).

A simple transportation network is shown in figure 1. Depots 1, 2, and 3 are the source nodes, Stations 1,2, ..., 5 are the destination nodes, the arcs between nodes represent the existence of a path. The supply and demand requirements appear beside the nodes. For example, depot 1 is capable of producing 89945 cubic-meters (M^3) of Gasoline and station 1 needs at least 3489 M^3 as shown in Figure 1 and 2.

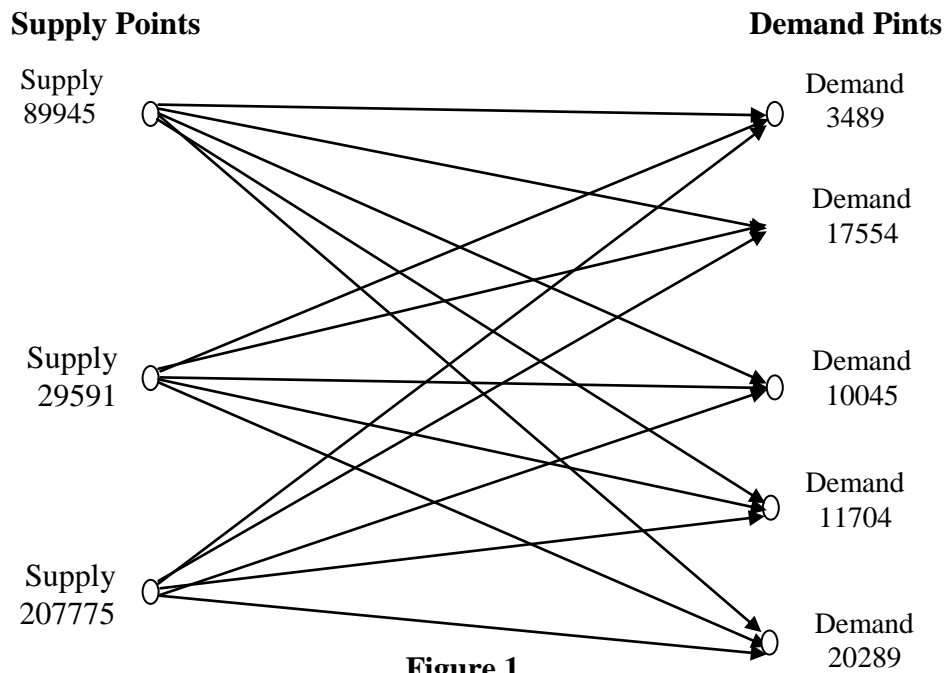


Figure 1

All the information can also be stored in a tableau form. Figure 2 shows the tableau form

Stations Depots	Station1	Station2	Station3	Station4	Station5	Supply
Depot 1	2340	2080	2795	4875	5200	89945
Depot 2	1755	1365	2795	2980	6175	29591
Depot 3	1105	1885	3510	6695	7020	207775
Demand	3489	17554	10045	11704	20289	

Figure 2

This problem is simple and can be solved without much effort. However, practical transportation networks are much more complicated and intelligent techniques are needed to solve them efficiently. One way to solve Transportation Problems is through a method called Linear Programming. The next section will briefly discuss how this can be done.

Mathematical Form of Linear Programming Problem

Linear Programming is the mechanism of maximizing or minimizing a linear function over a convex polyhedron. Linear Programming applies to optimization problems in which the objective and constraint functions are strictly linear. Therefore; the general standard form of the linear primal is defined as:

$$\left(\begin{array}{l} \text{maximize} \\ \text{or} \\ \text{minimize} \end{array} \right) Z = \sum_{j=1}^n c_j x_j$$

Subject to

$$\begin{array}{ll} \sum_{j=1}^n a_{ij} x_j = b_i, & i = 1, 2, 3, \dots, m \\ x_j \geq 0 & j = 1, 2, 3, \dots, n \end{array}$$

Solving a Transportation Problem as a LP

Transportation Problem can be modeled as a Linear Programming problem. Such that the amount of supply at source i is S_i and the demand at destination j is D_j . The unit transportation cost between source i and destination j is C_{ij} .

Let X_{ij} represent the quantities of the goods to be shipped from source (i) to destination (j); then the LP model representing the transportation problem is given generally as:

$$\begin{array}{ll} \text{Minimize:} & Z = \sum_{i=1}^m \sum_{j=1}^n C_{ij} X_{ij} \\ \text{Subject to} & \\ & \sum_{j=1}^n X_{ij} \leq S_i \quad ; \text{ For } i = 1, 2, 3, \dots, m \\ & \sum_{i=1}^m X_{ij} \geq D_j \quad ; \text{ For } j = 1, 2, 3, \dots, n \\ & X_{ij} \geq 0 \quad ; \text{ For all pairs } (i, j) \end{array}$$

Simplex method

The most general technique of LP is called the Simplex Method it uses a tableau form of representing the numbers. The algorithm has two basic parts: (1) Find an initial basic feasible solution, (2) Obtain an optimal

solution by making successive improvements to initial basic feasible solution until no further decrease in the transportation cost is possible. The Simplex Method is a greedy algorithm. It obtains basic feasible solutions by making the most improvement from the previous solution in every iteration (Sharrma, 1988).

Feasible Solution

A set of non-negative values X_{ij} , $i = 1,2,3,\dots,m$; $j = 1,2,3,\dots,n$, that satisfies the constraints is called a feasible solution to the transportation problem (Gupta & Hira, 2002).

Basic Feasible Solution

A feasible solution that contains no more than $(m + n - 1)$ non-negative allocations is called a basic feasible solution to the transportation problem (Gupta & Hira, 2002).

For example, the basic feasible solution for the problem in figure 2 by Vogel method is:

Stations Depots	Station1	Station2	Station3	Station4	Station5	Dummy station	Supply	
Depot 1	2340	2080	2795	4875	5200	89945	89945	
Depot 2	1755	1365	2795	11704	2980	6175	17887	29591
Depot 3	3489	17554	10045		20289		156398	207775
Demand	1105	1885	3510	6695	7020		264230	

Figure 3

Transportation Simplex method

The Transportation Simplex Method is a special version of the Simplex Method used to solve Transportation Problem. Although it has the basic steps as Simplex Method, it has a much more compact tableau form. This compact form takes less memory, therefore might be faster. For detailed steps of the Simplex and Transportation Simplex Method, see (Sharrma, 1988; Winston,1987)

Computational Difficulties

Any Linear Programming Problem can be solved by using the Simplex Algorithm. With the Transportation Simplex method, Transportation Problem can be solved accurately. However, it requires too much

computation time to solve large-scale problems with these methods. Transportation Problems are Integer Linear Programming Problems in essence. When the Simplex Method is used, a lot of intermediate points are found by the algorithm that eventually gets rejected. Therefore, it takes too long for these algorithms to solve TSPs. Since Lagrange Multiplier is a mathematical tool for solving constrained optimization of differentiable functions. As known, the Lagrange multipliers can be used to solve non-linear programming in which all the constraints are equality constraints. But hence, we try to use it to find optimality solution for linear transportation problem.

In this research project, we shall discuss the Lagrange's multipliers method which provides a necessary condition for an optimum when constraints are equations.

Lagrange Multipliers

In this research will cover how to find the maximum and minimum points on a function subject to constraint. This technique is called Lagrange's Method. Suppose we wish to find an optimal solution of a differentiable function. We consider linear programming of the transportation problem:

$$\left. \begin{aligned} \text{Minimize } Z &= f(X_{11}, X_{12}, \dots, X_{nm}) \\ &= \sum_{i=1}^m \sum_{j=1}^n C_{ij} X_{ij} \\ \text{whose variables are subject to the } (m+n) \text{ constraints} \\ &\sum_{j=1}^n X_{ij} = S_i \text{ for } i = 1, 2, 3, \dots, m \\ \text{and} \\ &\sum_{i=1}^m X_{ij} = D_j \text{ for } j = 1, 2, 3, \dots, n \\ &X_{ij} \geq 0 \quad ; \text{ for all } i \text{ and } j \end{aligned} \right\} \dots(1)$$

Since the transportation problem has $(m+n)$ equality constraints and (mn) inequality (non-negativity) constraints, then the Lagrange formulation must have one Lagrange multiplier for each of these constraints. These will now be defined. Let the Lagrange multiplier $(\alpha_i \in R^m)$ be associated with the equality constraint up on the amount (S_i) of supply at source (i) . The Lagrange will have (m) such terms, one for each (i)

$$\alpha_i [\sum_{j=1}^n X_{ij} - S_i] \quad ; \text{ for } i = 1, 2, 3, \dots, m$$

Let the Lagrange multiplier $(\beta_j \in R^n)$ be associated with the equality constraint up on the requirement (D_j) at destination (j) . The Lagrange will have (n) such terms, one for each (j) .

$$\beta_j[\sum_{i=1}^m X_{ij} - D_j] ; \text{ for } j = 1,2,3,\dots,n$$

Let the Lagrange multiplier ($\gamma_{ij} \in R^{m+n}$) be associated with the inequality constraint ($X_{ij} \geq 0$). The Lagrange will have (mn) such term

$$\gamma_{ij}(X - 0) \quad ; \text{ for each } i \text{ and } j$$

The Lagrange function, if all constraints are required to hold as equalities, is

$$L(X_{11}, X_{12}, \dots, X_{mn}, \alpha_1, \alpha_2, \dots, \alpha_m, \beta_1, \beta_2, \dots, \beta_n, \gamma_{11}, \gamma_{12}, \dots, \gamma_{mn}) = F(X, \alpha, \beta, \gamma)$$

$$F(X, \alpha, \beta, \gamma) = \sum_{i=1}^m \sum_{j=1}^n C_{ij} X_{ij} - \sum_{i=1}^m \alpha_i [\sum_{j=1}^n X_{ij} - S_i] - \sum_{j=1}^n \beta_j [\sum_{i=1}^m X_{ij} - D_j] - \sum_{i=1}^m \sum_{j=1}^n \gamma_{ij} X_{ij} \quad \dots(2)$$

involving the Lagrange multipliers ($\alpha_1, \alpha_2, \dots, \alpha_m, \beta_1, \beta_2, \dots, \beta_n, \gamma_{11}, \dots, \gamma_{mn}$). Then we attempt to find at least (theoretically) for ($2mn + n + m$) unknowns ($\bar{X}_{11}, \bar{X}_{12}, \dots, \bar{X}_{mn}, \bar{\alpha}_1, \bar{\alpha}_2, \bar{\alpha}_3, \dots, \bar{\alpha}_m, \bar{\beta}_1, \bar{\beta}_2, \bar{\beta}_3, \dots, \bar{\beta}_n, \bar{\gamma}_{11}, \bar{\gamma}_{12}, \dots, \bar{\gamma}_{m-1}, \bar{\gamma}_{mn}$), that minimize $F(X, \alpha, \beta, \gamma)$

If $(\bar{X}_{11}, \bar{X}_{12}, \dots, \bar{X}_{mn}, \bar{\alpha}_1, \dots, \bar{\alpha}_m, \bar{\beta}_1, \dots, \bar{\beta}_n, \bar{\gamma}_{11}, \bar{\gamma}_{12}, \dots, \bar{\gamma}_{mn})$ minimizes L , then at $(\bar{X}_{11}, \bar{X}_{12}, \dots, \bar{X}_{mn}, \bar{\alpha}_1, \dots, \bar{\alpha}_m, \bar{\beta}_1, \dots, \bar{\beta}_n, \bar{\gamma}_{11}, \bar{\gamma}_{12}, \dots, \bar{\gamma}_{mn})$

$$\left. \begin{aligned} \frac{\partial L}{\partial X_{ij}} &= 0 && ; \text{ for all } (i, j) \\ \frac{\partial L}{\partial \alpha_i} &= 0 && ; \text{ for } i = 1,2,3,\dots,m \\ \frac{\partial F}{\partial \beta_j} &= 0 && ; \text{ for } j = 1,2,3,\dots,n \\ \frac{\partial F}{\partial \gamma_{ij}} &= 0 && ; \text{ for all } (i, j) \end{aligned} \right\} \dots(3)$$

Here (3) is the partial derivative of the Lagrange function with respect to each variable. This shows that $(\bar{X}_{11}, \bar{X}_{12}, \dots, \bar{X}_{mn})$ will satisfy the constraints in (1).

We know that for $(\bar{X}_{11}, \bar{X}_{12}, \dots, \bar{X}_{mn}, \bar{\alpha}_1, \dots, \bar{\alpha}_m, \bar{\beta}_1, \dots, \bar{\beta}_n, \bar{\gamma}_{11}, \bar{\gamma}_{12}, \dots, \bar{\gamma}_{mn})$ to solve the Lagrange function, it is necessary that at $(\bar{X}_{11}, \bar{X}_{12}, \dots, \bar{X}_{mn}, \bar{\alpha}_1, \dots, \bar{\alpha}_m, \bar{\beta}_1, \dots, \bar{\beta}_n, \bar{\gamma}_{11}, \bar{\gamma}_{12}, \dots, \bar{\gamma}_{mn})$

$$\frac{\partial L}{\partial X_{11}} = \frac{\partial L}{\partial X_{12}} = \dots = \frac{\partial L}{\partial X_{nm}} = \frac{\partial L}{\partial \alpha_1} = \dots = \frac{\partial L}{\partial \alpha_m} = \frac{\partial L}{\partial \beta_1} = \dots = \frac{\partial L}{\partial \beta_n} = \frac{\partial L}{\partial \gamma_{11}} = \frac{\partial L}{\partial \gamma_{12}} = \dots = \frac{\partial L}{\partial \gamma_{nm}} = 0 \quad \dots(4)$$

The following theorem gives conditions, which imply that any point $(\bar{X}_{11}, \bar{X}_{12}, \dots, \bar{X}_{mn}, \bar{\alpha}_1, \dots, \bar{\alpha}_m, \bar{\beta}_1, \dots, \bar{\beta}_n, \bar{\gamma}_{11}, \bar{\gamma}_{12}, \dots, \bar{\gamma}_{mn})$ that satisfies (4) will yield an optimal solution $(\bar{X}_{11}, \bar{X}_{12}, \dots, \bar{X}_{mn})$ to (1).

Theorem: (Boyd & Vandenberghe, 2004)

Suppose that:

Minimize: $Z = f(X_1, X_2, \dots, X_n)$

Subjected to: $g_1(X_1, X_2, \dots, X_n) = b_1$

:

$g_m(X_1, X_2, \dots, X_n) = b_m$

If $f(X_1, X_2, \dots, X_n)$ is convex function and each $g_i(X_1, X_2, \dots, X_n)$ is a linear function then any point $(\bar{X}_1, \bar{X}_2, \dots, \bar{X}_n, \bar{\alpha}_1, \dots, \bar{\alpha}_m)$ satisfies (4) will yield an optimal solution $(\bar{X}_1, \bar{X}_2, \dots, \bar{X}_n)$ to above equation Z.

Theorem: (Dano, 1975)

In minimization problem, if $(X_0 = x'_1, x'_2, \dots, x'_n)$ is an optimal solution of

$$f(x_1, x_2, \dots, x_n)$$

Subject to

$$g_1(x_1, x_2, \dots, x_n) \geq b_1$$

:

$$g_m(x_1, x_2, \dots, x_n) \geq b_m$$

$$x_1 \geq 0$$

:

$$x_n \geq 0$$

, then $(X_0 = x'_1, x'_2, \dots, x'_n)$ must satisfy the (m) constraints, and there must exist multipliers $\alpha_1, \alpha_2, \dots, \alpha_m, \gamma_1, \gamma_2, \dots, \gamma_n$ satisfying

$$\frac{\partial f(X_0)}{\partial x_j} + \sum_{i=1}^m \alpha_i \frac{\partial g_i(X_0)}{\partial x_j} - \gamma_j = 0 \quad ;(j = 1, 2, 3, \dots, n)$$

$$\alpha_i [b_i - g_i(X_0)] = 0 \quad ;(i = 1, 2, 3, \dots, m)$$

$$\alpha_i \geq 0 \quad ;(i = 1, 2, 3, \dots, m)$$

$$\gamma_j \geq 0 \quad ;(j = 1, 2, 3, \dots, n)$$

The Kuhn-Tucker Condition in Transportation Problem

In Lagrange multiplier method, it becomes difficult to solve the system of (mn) simultaneous equations. This difficulty can be removed by using Kuhn-Tucker Conditions. This research is concerned with developing the necessary and sufficient conditions for identifying the basic feasible solution of the general inequality constrained optimization transportation problems. These conditions are called the Kuhn-Tucker conditions. The development is based mainly on Lagrange method.

The second theorem give conditions that are necessary for a point $(X_0 = x'_1, x'_2, \dots, x'_n)$ to be an optimal solution to function $f(x_1, x_2, \dots, x_n)$ which

is called the Kuhn tucker conditions (Dano,1975). Now, since the transportation problem is minimization problem, the Kuhn-tucker conditions are thus given by:

i. The derivative of $F(X, \alpha, \beta, \gamma)$ with respect to (X_{ij}) is:

$$\frac{\partial F}{\partial X_{ij}} = C_{ij} - \alpha_i - \beta_j - \gamma_{ij} = 0 \quad ; \text{ for all } (i) \text{ and } (j)$$

This given (mn) derivative equations, and

$$\gamma_{ij} = C_{ij} - \alpha_i - \beta_j \quad ; \text{ for all } (i) \text{ and } (j)$$

ii. We have:

$$\alpha_i [\sum_{j=1}^n X_{ij} - S_i] = 0 \quad ; \text{ for } i = 1, 2, 3, \dots, m$$

$$\beta_j [\sum_{i=1}^m X_{ij} - D_j] = 0 \quad ; \text{ for } j = 1, 2, 3, \dots, n$$

$$\gamma_{ij} X_{ij} = 0 \quad ; \text{ for all } (i) \text{ and } (j)$$

iii. Since the objective function is minimum and (γ_{ij}) associated with the inequality (non-negative) constraints, then

$$\gamma_{ij} \geq 0 \quad ; \text{ for all } (i) \text{ and } (j)$$

iv. Lagrange multipliers corresponding to equality constraints must be unrestricted in sign.

It is clear from (ii) that not all non-negativity constraints can hold as equalities, not all shipments can be zero. For each (X_{ij}) allowed to be positive, the corresponding (γ_{ij}) will be set to zero. Therefore;

$$C_{ij} - \alpha_i - \beta_j = 0 \quad ; \text{ for each } (i, j) \text{ pair such that}$$

$$X_{ij} > 0$$

Further, for each zero shipment cell $\gamma_{ij} \geq 0$.

The method determines the (γ_{ij}) values by determining first (α_i) and (β_j) values. There are (m) sources and thus (m) α_i values to be determined. There are (n) destinations and (n) β_j values to be determined. Hence there is a total of $(m+n)$ unknowns.

The conditions that determine these unknowns are based upon the requirement that $\gamma_{ij} = 0$ for all cells of the current solution. Assuming that the current solution is not degenerate, there are $(m+n-1)$ non-zero shipment cells whose $\gamma_{ij} = 0$. For those cells the $(m+n-1)$ equation

$$C_{ij} = \alpha_i + \beta_j \quad ; \text{ for all } (i) \text{ and } (j)$$

must hold.

Calculate (γ_{ij}) for each cell (i, j) by using the formula

$$\gamma_{ij} = C_{ij} - \alpha_i - \beta_j \quad ; \text{ for all } i \text{ and } j$$

Therefore; if $\gamma_{ij} \geq 0$ (which implies increase in cost) for each zero shipment cell, then the basic feasible solution under test must be optimal. Otherwise, if $\gamma_{ij} < 0$ (because negative difference implies decrease in cost) for one or more zero shipment cells, then it would be better to reduce the cost more by allocating as much as possible to the cell with the largest negative (smallest) value of γ_{ij} . This way, it is possible to improve the basic feasible solution successively for reduced cost till the optimal solution is obtained for which $\gamma_{ij} \geq 0$ for each zero shipment (empty cell).

It is well known that in pure transportation problem, there will be at most $(m+n-1)$ positive source-destination flows in an optimal solution to the cost-minimizing problem. Thus, the system of Lagrange constraints which hold with equality will have $(m+n-1)$ equations in $(m+n+mn)$ unknowns. One of the Lagrange multipliers can be normalized to zero, with all other Lagrange multipliers calculated relative to this anchor point. This result also implies that there will be most $(m-1)$ sources, which supply more than one destination.

We noted that, the method of Lagrange multipliers reproduces exactly the same iterations as the multipliers method, but it reduces the time in searching the optimal solution. The main difference occurs in the manner in which the multipliers method is based on duality theory. The development of the Lagrange multipliers method is based on solving the problem with non-linear structure.

Solving Transportation Problems Using Lagrange Multiplier Algorithm

To solve Transportation Problems using this Lagrange method, the following steps are needed:

1. Forming the Fitness Lagrange Function. Forming the Fitness Lagrange Function is generated by using the equation 2. For example, the objective function for problem in figure 2 is:

Forming the Lagrange function L, for the Gasoline product, we begin with basic feasible solution by using Vogel method in figure 3, then:

$$\begin{aligned} L(X, \alpha, \beta, \gamma) &= \sum_{i=1}^m \sum_{j=1}^n C_{ij} X_{ij} - \sum_{i=1}^m \alpha_i (\sum_{j=1}^n X_{ij} - S_i) - \sum_{j=1}^n \beta_j (\sum_{i=1}^m X_{ij} - D_j) - \sum_{i=1}^m \sum_{j=1}^n \gamma_{ij} X_{ij} \\ &= \sum_{i=1}^3 \sum_{j=1}^5 C_{ij} X_{ij} - \sum_{i=1}^3 \alpha_i (\sum_{j=1}^5 X_{ij} - S_i) - \sum_{j=1}^5 \beta_j (\sum_{i=1}^3 X_{ij} - D_j) - \sum_{i=1}^3 \sum_{j=1}^5 \gamma_{ij} X_{ij} \\ &= 2080X_{24} + 1105X_{31} + 1885X_{32} + 3510X_{33} + 7020X_{35} - [\alpha_1(X_{16} - 89945) \\ &+ \alpha_2(X_{24} + X_{26} - 29591) + \alpha_3(X_{31} + X_{32} + X_{33} + X_{35} + X_{36} - 207775)] - \beta_1(X_{31} - 3489) \\ &- \beta_2(X_{32} - 17554) - \beta_3(X_{33} - 10045) - \beta_4(X_{24} - 11704) - \beta_5(X_{35} - 20289) \\ &- \beta_6(X_{16} + X_{26} + X_{36} - 264230) \end{aligned}$$

2. The next step; differentiating $L(X, \alpha, \beta, \gamma)$ with respect to each variable and setting the equal to zero, we obtain

$$\frac{\partial F}{\partial X_{ij}} = C_{ij} - \alpha_i - \beta_j - \gamma_{ij} = 0 \quad ; \text{ For all } i=1, 2, 3 \text{ and } j=1, 2, 3, 4, 5$$

By letting $(\alpha_1 = 0)$ the values of Lagrange multipliers are successively determined, then computing $\gamma_{ij} = C_{ij} - \alpha_i - \beta_j$ for each non-basic variable. Thus the optimal solution for transportation problem is finding after 5th iterations.

Conclusions

In comparison to the existing methods, Lagrange algorithm proves to be more efficient as the size of the problem becomes greater for a problem. The non-linear method is much faster, easier and shorter than the simplex and transportation simplex method. This solution is not computationally very fast. We are interested in identifying the conditions under which the objective function associated with an optimal solution of the transportation problem is equal to the bound given by maximizing (minimizing) the expected overlap.

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تطبيق مضاعف لكرانج للبرمجة الخطية الصحيحة في الانتاج- النقل مع كلفة النقل المرنة

عبدالرحيم خلف راهي سوزان صابر حيدر

كلية الادارة و الاقتصاد – جامعة السليمانية

تاريخ الاستلام: ٢٠/٦/٢٠٠٧، تاريخ القبول: 2008/4/8

الخلاصة

(أن مشكلة النقل هي احدى المشاكل التقليدية لبحوث العمليات حيث ان الهدف منها نقل السلع من مصادر تصنيعها او من المخازن الى مراكز متعددة بهدف سد حاجة هذه المراكز بأقل كلفة ممكنة . و كما ان مشكلة النقل يمكن ان تحل كمشكلة برمجة خطية. والبرمجة الخطية بصورة عامة تستعمل فيها الطريقة المبسطة (Simplex Method) لحل المسائل، حيث استخدمت الخوارزمية لحل مسائل البرمجة الخطية بالتقدم من نقطة متطرفة واحدة من الشكل المتعدد السطوح إلى نقطة مجاورة. حيث تم في هذه الدراسة حل مشكلة النقل من خلال صياغتها بشكل نموذج رياضي ومقارنة نتائجها مع نتائج استخدام البرمجة اللاخطية، المتمثلة بطريقة مضاعفات لكرانج. ان الاخيرة تستعمل الآليات المختلفة لإختيار أفضل الحلول المثلى. حيث تستند على تحويل مشكلة النقل ذو التركيب الخطي إلى التركيب اللاخطي ومن ثم حلها باستعمال تقنيات البرمجة اللاخطية مباشرة، بدلا من التقنيات الخطية. الهدف من البحث هو معرفة الية تحقيق خوارزمية لكرانج من ناحية الدقة والسرعة عند حل مشكلة نقل واسعة النطاق.