A Review of Peak to Average Power Ratio Reduction Schemes in OFDM Systems

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Abstract

This paper is concerned with the Orthogonal Frequency Division Multiplexing (OFDM) system that is adopted by a wireless communication system for transmitting multicarrier modulated signals simultaneously with robustness against interference and noise facing the system. It presents a significant issue related to the OFDM system known as the raise of Peak to Average Power Ratio (PAPR) that is the high peaks of OFDM signals compared to its average power. Increasing the PAPR yields in the component devices of the OFDM system does not cope with the high peaks resulting in distortion and signal losses. Therefore, the objective of this article is to review different approaches of PAPR reduction techniques and compare them to select the best technique for a particular system. The approaches are mainly of three classifications; distortion, probabilistic, and coding schemes with each including several techniques.

1. Introduction:

In wireless communication systems nowadays, services of high data rates are in high demands. Therefore, very reliable data transmission over very harsh environments is required. On the other hand, these transmission systems face many impairments such as large attenuation, noise, multipath fading, interference, etc. As a consequence, a robust technique for implementing the system is desired that can deal with such impairments, known as multi-carrier modulation technique, OFDM is the most common technique used in practice [1] [2]. OFDM is a multi-carrier orthogonal digital communication scheme in which the available bandwidth is divided into many parts of low data rates and each part is modulated simultaneously by multi-carriers. Adding up the individual modulated signals yields in a problem called PAPR, the ratio of maximum power to the average power for a given signal. Increasing PAPR results in Digital-to-Analog Convertor (DAC) and High-Power Amplifier (HPA) requiring a larger dynamic range to prevent signal distortion (avoid the amplitude clipping). This leads to higher power consumption and component costs of the transceiver. That is the reason for decreasing the amount of PAPR [3] [4] [5]. There are various approaches to reduce PAPR based on three main classified techniques, which are OFDM signal distortion, probability of getting high peaks of the OFDM signal, and coding that implies set of code words to put onto the original OFDM data [3]. Or it can be classified into two categories according to the distortion; distortion signals and distortionless techniques [6]. The objective of this paper is to review those approaches and provide advantages and disadvantages of each approach then making comparisons among them. The paper is ordered as, introduction as the first section, a brief background on OFDM and PAPR in the second section. Then a deep insight into the techniques for PAPR reduction are presented and discussed in the third section. Finally, the criteria for selecting appropriate PAPR reduction approach is in the fourth section.

2. Background on OFDM and PAPR:

2.1 The basic principle of OFDM:

OFDM is a modified version of Frequency Division Multiplexing (FDM). The mechanism of OFDM is to make the subcarriers of FDM orthogonal through the use of Inverse Fast Fourier Transform (IFFT). The orthogonality is advantageous in reducing interference and eliminating the need for guard bands. As a result, the OFDM becomes reliable and spectrum efficient and these characteristics make it widely used in wireless communication and broadcasting. Figure 1,
represents the transmitter of the OFDM system. As it can be seen from Figure 1, OFDM can be viewed as a modulation technique or multiplexing technique. The information bits can be in binary or in random data whereas in the binary, M-ary Phase Shift Keying (M-PSK) is used for modulation while for random data, Quadrature Amplitude Modulation (QAM) is used. After modulating the data, symbols are converted to parallel stream by the Serial to Parallel (S/P) convertor. Then each block of parallel data ($X_k$) is superimposed on an orthogonal carrier and converted from frequency domain to time domain ($X_n$) by the IFFT operation.

$$X_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{\frac{j2\pi k n}{N}}$$  

Where, $0 \leq n \leq N-1$ and $k$ is the number of subcarriers. Making the frequency spacing between each successive subcarriers ($\Delta f$) an integer multiple of reciprocals of bits time duration ($T_s$), keeps the orthogonality of the subcarriers.

$$\Delta f = \frac{k}{T_s}$$

Where, $k \in I$. After that, Cyclic Prefix (CP) is added to the data that is to prevent Intersymbol Interference (ISI). The data is converted back to serial form with the Parallel to Serial (P/S) convertor. Finally, the data is transmitted after power amplification over a wireless channel such as Additive White Gaussian Noise (AWGN), Rayleigh, and Rician. In the receiver side, the operations of the transmission are repeated but in reverse order. Figure 2 represents the receiver of the OFDM system.

2.2 PAPR:

PAPR as the name suggests, is the ratio of maximum power to the average power of the transmitted signal.

$$PAPR = \frac{\max(|X_n|^2)}{E(|X_n|^2)}$$

Where, $E(|X_n|^2)$ denotes the mean value.

The subcarriers in OFDM are independent. Therefore, different signals may have high peaks (maximum peaks) simultaneously resulting in a high PAPR. In case a small dynamic range of Power Amplifier (PA) for the OFDM transmission system is provided, the high amplitudes are saturated leading to distortion. However, extending the dynamic range of PA requires very high input energy. Hence, a trade-off between distortion and power consumption exists. Therefore, PAPR reduction is considerably studied by researchers [10][11][12][13][14]. Referring to the PAPR formula, two options can be made in order to minimize the PAPR, either by reducing the maximum peak and/or by increasing the average power (mean value). But boosting the latter will require more power consumption. Therefore, the former is chosen. Furthermore, reducing PAPR before transmission simplifies the design of PA and reduces the effects of distortion [7]. The next section provides different approaches of PAPR reduction.

3. PAPR Reduction Approaches:

The approaches for reducing PAPR can be classified mainly into three categories, signal distortion, probabilistic, and coding techniques. As shown in Figure 3. This section provides an overview on some of these techniques with their advantages and disadvantages.

3.1 Signal Distortion Techniques:

Signal distortion techniques include, Clipping and Filtering, Peak Windowing, and Comping Transforms. They are
based on distorting the OFDM signal before the PA thus reducing the PAPR to an acceptable level but at the cost of signal distortion.

### 3.1.1 Clipping and Filtering:
A very basic approach for reducing PAPR is through clipping. A clipper is utilized to limit the high peaks of the input signal by setting a predetermined value to cut off the exceeded values while allowing the lower-level signals to pass. This can be represented mathematically as:

\[
B(x) = \begin{cases} 
  x & |x| \leq A \\
  A e^{j\phi(x)} & |x| > A 
\end{cases}
\]

(4)

Where \( x \) is the OFDM signal, \( \phi(x) \) is the phase of the signal, \( A \) is the level predetermined by the clipper, and \( B(x) \) is the signal after the clipping process. Due to the nonlinearity of clipping, the approach has two drawbacks which are in-band distortion and out-of-band radiation. They degrade the system performance such as Bit Error Rate (BER) and spectral efficiency. Therefore, for a better system performance, reducing/removing the out-of-band radiation, and eliminating noise, filtering of the clipped signal is used. Nevertheless, this technique may cause high peaks to regrow and once again results in increasing the PAPR. For fixing this issue, the clipping and filtering need to be repeated until the required level of PAPR is achieved. This method refers to Iterative Clipping and Filtering (ICF). The amount of repetition on the other hand, causes high complexity. Recently, attempts have been made toward decreasing the number of iterations. Figure 4 shows the architecture of this approach clearing out the process of iteration through the use of feedback connection, connecting the output signal to the input of the clipper when the high peak values exceed the value determined by the clipper. Furthermore, the order of the process is shown that is clipping before the filtering and then to IFFT [15] [16].

![Figure 4. Iterative clipping and filtering architecture for PAPR reduction [15].](image)

### 3.1.2 Peak Windowing:
This approach reduces the high peaks of the OFDM signal by inserting a windowing function to the system. The function of the window is to multiply the original OFDM signal with a finite length window signal that varies in amplitude. The principle of such a mechanism is to multiply the high peaks of the OFDM signal by the minimum peaks of the window signal while multiplying its valleys to the maximum peaks of the window signal. Hence, the method suppresses out-of-band radiation and offers more smooth edges compared to the clipping method. However, as the number of high peaks increases, the peak reduction worsens [17]. Different window functions can be used to implementing this technique depending on the signal and suitability such as Hamming, Hanning, and Kaiser windows. Their mathematical expressions are provided in the following:

\[
\text{Hamming: } W_{\text{Hamm}} = \begin{cases} 
  0.54 + 0.46\cos\left(\frac{2\pi n}{N-1}\right), & \text{for } 0 \leq n \leq N-1 \\
  0, & \text{otherwise}
\end{cases}
\]

(5)

\[
\text{Hanning: } W_{\text{Hann}} = \begin{cases} 
  0.5 + 0.5\cos\left(\frac{2\pi n}{N-1}\right), & \text{for } 0 \leq n \leq N-1 \\
  0, & \text{otherwise}
\end{cases}
\]

(6)

\[
\text{Kaiser: } W_{\text{kaiser}} = \begin{cases} 
  I_0(\pi n)\sqrt{1-\left(\frac{2n}{N-1}\right)^2}, & \text{for } 0 \leq n \leq N-1 \\
  0, & \text{otherwise}
\end{cases}
\]

(7)

Where in both functions, \( n \) represents the number of times repeating the equation, and \( N \) is the width in samples of a discrete-time of window function.

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  0, & \text{otherwise}
\end{cases}
\]

(7)

Where in both functions, \( n \) represents the number of times repeating the equation, and \( N \) is the width in samples of a discrete-time of window function.

### 3.1.3 Companding transform:
Companding transform is considered as the most attractive approach for PAPR reduction. It has the advantage of slight computational complexity because it is not affected by the number of subcarriers of OFDM. Moreover, the amount of bit rate does not suffer losses because the side information does not require to be transmitted. The companding transform is in fact an optimized version of the clipping technique. The major difference between them is that clipping approach only deals with the high peaks (terminating the high peaks) without considering the lower amplitudes and as a consequence of this, in-band distortion and out-of-band radiation occurs. While the companding transform compresses the high peaks and enlarges the lower amplitude signals in order to reject any interference coming from noise. Moreover, regarding the clipping technique when the high peaks are clipped the receiver cannot recover the original OFDM signal. However, the
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3.2 Probabilistic Techniques:
Probabilistic techniques concentrate on the probability of appearing high peaks of OFDM signals, depending on the low probability of appearance. The idea is to supply a number of scrambling sequences to the OFDM signal to boost the OFDM phase sequences. Then the combined OFDM signal and phase sequence that has the least PAPR is chosen to be transmitted. Therefore, the probability of occurring high peaks is reduced thus, the PAPR is reduced.

3.2.1 Selective Mapping:
It is a distortionless PAPR reduction approach involving the generation of a sequence of different candidate data vectors. When transmitting each data vector, the lowest PAPR must be selected by rotating the phase of every symbol data. A set of D fixed phase rotation but at the same time different, distinct, and pseudorandom data for the Selective Mapping (SLM) is provided as follows;

\[ P^{(d)} = [p_{0}^{(d)}, p_{1}^{(d)}, \ldots, p_{N-1}^{(d)}] \]

\[ P_{k}^{(d)} = e^{j \phi_{k}^{(d)}}, \phi_{k}^{(d)} \in [0, 2\pi] \]

And k=0,1,\ldots,N-1,d=1,2,\ldots,D
Where every parameter must be defined and available at both transmitter and receiver. The data vector S\( \mu \) is element-wisely multiplied by each of the phase rotation vectors P\( ^{(d)} \). Thus, obtaining a set of different phase rotated data precises as follows;

\[ s(\mu, d) = S^{\mu} P^{(d)} \]

After that IFFT is applied to the D data vectors to be converted to time domain signals and selected as candidates for the OFDM signal. The mathematical expression is presented as;

\[ s_{n/L}^{(\mu, d)} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_{k}^{(\mu)} P_{k}^{(d)} e^{j \frac{2\pi}{N} kn}, n = 0, 1, \ldots, LN - 1 \]

\[ s_{n/L}^{(\mu, d)} = IFFT[\sqrt{L}s_{L}^{(\mu, d)}] \]

The transmitter chooses among the D candidates one with lowest PAPR, as shown in the following equation:

\[ \hat{d} = \arg \min_{1 \leq d \leq D} \text{PAPR}(s_{n/L}^{(\mu, d)}) \]

For better understanding of the principle of operation, the following scheme provides the architecture of the SLM technique [20][21].

Figure 5. SLM- OFDM transmitter architecture [20].

3.2.2 Partial Transmit Sequences (PTS):
Explaining the principle of operation for this technique is clarified with the help of the block diagram of Figure 6. The data vector X of k information symbols is applied to the partition
Where, \( v = 0, 1, \ldots, V - 1 \)

After the partitioning, another operation of oversampling by \( L \) factor is executed on each subblock vector. Then these \( LK \)-length subblocks are inserted to the IFFT.

\[
x^{(v)} = [x_0^{(v)}, x_1^{(v)}, \ldots, x_{LK-1}^{(v)}] = \text{IFFT}[X^v]
\]

Where, \( 0 \leq v \leq V - 1 \)

The phase factor optimization block is the next step after the IFFT process in which the time domain subblock vectors obtained from the output of IFFT are multiplied by a phase factor.

\[
b_v = e^{j\phi}, \phi \in [0, 2\pi)
\]

Where, \( v = 0, 1, \ldots, V - 1 \)

Finally, the related subblock vectors are added up throughout rotating the phase of each subblock. The key objective of this approach is to find a proper combination of the phase factors that minimizes the PAPR [23][24].

### 3.2.3 Tone Reservation (TR):

In this approach, a small number of subcarriers of the OFDM block are reserved to transmit subcarriers that can reduce the PAPR of the OFDM signal. The principle of such algorithm is to create a signal with distinct frequency domain features and transmitting it over the reserved subcarriers then putting them all together with the useful remained subcarriers, in the frequency domain. The combined TR subcarriers with constructed signal tend to reduce the high peaks of the useful subcarriers and as a consequence reducing the PAPR.

Furthermore, the created signal at the receiver side can be easily removed if the receiver does not require any side band information. Which is the case in this technique since the original signal and the added signal are structured in a way each complete the other. With the assumptions that the original signal to be transmitted is \( X \) in the frequency domain and \( C \) is also a frequency domain additional signal, to offer the PAPR reduction, \( N \) is the total number of subcarriers, and the TR subcarriers can be denoted by \( R \), as shown in Figure 7 [25][26].

### 3.2.4 Tone Injection (TI):

The TI method reduces PAPR by expanding the constellation of the modulation technique, the M-ary QAM modulation may be used, through mapping each of its points into several different constellation points, and these points are able to carry information. Some symbols of the expanded constellation can substitute some of the original OFDM signal that is required to reduce PAPR. Furthermore, a point of the expanded constellation can replace a point of the original constellation, which is the same principle as injecting a tone of suitable phase and frequency to the original OFDM signal [27].

#### 3.2.5 Active Constellation Extension (ACE):

This approach reduces the PAPR by extending the constellation vector. Taking into consideration a noise model for the channel such as AWGN, the modulation symbol is not detected properly at the receiver side. In case the original signal to be transformed by the noise component is out of range of the probable-decision fields. These points out the fact that any point farther from the field of the decision will not enlarge the error rate. Therefore, careful designing of the extension vector can terminate the time domain high peaks of the OFDM symbol.

### 3.3 Coding Techniques:

The name implies that this technique deals with codes for gaining the reduction of PAPR. Fundamentally, a set of code words is chosen having a low PAPR feature then the original OFDM signal is mapped into the code words set so that the high peaks of the OFDM data symbol are dissipated by the mapping process. The algorithm for implementing this approach is simple and distortionless. Therefore, it is a preferable and well-known method among the other approaches for the PAPR reduction. Various methods for the mapping scheme are offered by authors such as; block coding, Reed Solomon, Hamming, Golay sequence, and low-Density Parity Check codes [28].

#### 3.3.1 Block Coding:

The principle of block mapping starts with a simple Linear Block Coding (LBC) that is mapping k-bit data words into n-bit code words by adding n-k parity bits. Then it has been developed to a method of mapping that can be applied to any number of subcarriers that is referred to Cyclic Coding (CC) and it utilizes the PAPR reduction by more than 3 dB. Later on, the method dealt with subblocks instead of bit streams known as Subblock Coding (SBC). In this technique, the long blocks of data are divided into many subblocks with an added parity bit to each subblock. The degree of PAPR reduction depends...
Table 1. Comparison of PAPR reduction approaches in terms of their criteria.

<table>
<thead>
<tr>
<th>PAPR reduction approaches</th>
<th>BER Increase</th>
<th>Bit rate Loss</th>
<th>Complexity</th>
<th>Power consumption increase</th>
<th>Bandwidth expansion</th>
<th>Distortion-less</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clipping</td>
<td>yes</td>
<td>no</td>
<td>low</td>
<td>No</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Companding</td>
<td>yes</td>
<td>no</td>
<td>low</td>
<td>No</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>SLM</td>
<td>no</td>
<td>yes</td>
<td>high</td>
<td>No</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>PTS</td>
<td>no</td>
<td>yes</td>
<td>high</td>
<td>No</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>TR</td>
<td>no</td>
<td>no</td>
<td>high</td>
<td>Yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>TI</td>
<td>no</td>
<td>No</td>
<td>high</td>
<td>Yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>ACE</td>
<td>no</td>
<td>no</td>
<td>high</td>
<td>Yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Coding</td>
<td>yes</td>
<td>yes</td>
<td>high</td>
<td>No</td>
<td>yes</td>
<td>no</td>
</tr>
</tbody>
</table>

3.3.2 Reed Solomon Codes:
These are non-binary cyclic error correcting codes. The basic operation of the mapping process is that the set of code words in this type are polynomials and are constructed from the original data symbol to be transmitted and then instead of sending the original data, the over-sampled polynomial is sent. At the receiver side, it can be easily reconstructing the polynomial even with a certain amount of transmission errors due to the redundancy of information contained in the over-sampled polynomial. Thus, applying a low PAPR set of polynomial code words yields in reducing the main PAPR of the original OFDM data [29] [30].

3.3.3 Golay Sequences:
Golay sequences are utilized for PAPR reduction due to their characteristics of good autocorrelation, since the sum of autocorrelations of a golay sequence is zero except at zero shift. The golay sequence offers limits for the PAPR hence, eliminating the higher peaks of the boundary [31] [32].

4. Criteria for Selecting Appropriate PAPR Reduction Approach:
Each approach is characterized by distinct characteristics and some common features among them. In accordance to these features and adjustability, different approaches are selected for different practical systems. A particular system might demand one or a hybrid between two or more approaches for satisfying the implementation. The following factors must be taken into consideration when choosing an approach. The most important and fundamental factor is the PAPR reduction level, it must be as large as possible but its effects on the other factors must be considered. The complexity of a system is another factor. Normally, PAPR degrades with more complex computational techniques on the other hand however, complexity requires additional time, hardware, and power consumption that is not desirable. In terms of bandwidth, some techniques require side information to be transmitted and that needs additional bandwidth. However, bandwidth cannot be easily obtained. Therefore, it is preferred to reduce PAPR but not in the cost of bandwidth. Another factor is the BER. When attempting to reduce PAPR, the system performance needs to be efficient that includes sufficient BER. There are other factors for instance, digital to analog convertor, HPA, and mixer are nonlinear devices capable of reducing PAPR without nonlinear distortion. Nonetheless, the cost of these devices must be taken into account.

Table 1, compares and summarizes the PAPR reduction approaches discussed in this article corresponding to their characteristics. It can be observed that no particular approach is perfect having all the criteria but, depending on the practical systems suitable techniques are used [3].

5. Conclusion:
The article presented an overview of several approaches regarding the PAPR reduction techniques in the OFDM system. Despite that OFDM acts as a key element in wireless communication due to its reliability and immunity against interference and noise still, it faces a major problem that is the addition of individual subcarriers results in high peaks of OFDM signal, and this results in the disability of OFDM system devices to cope the high peaks. Therefore, the approaches discussed in the article offered methods for reducing PAPR. The techniques are mainly classified into three main methods based on distortion, probability, and coding schemes. With each having different approaches with their own features, advantages, and disadvantages. A comparison has been made after explaining each of them individually to offer a better insight in selecting the most appropriate technique. It
was concluded from the comparison that every technique has imperfections. Therefore, according to the demands of a particular practical system a technique is selected. The study of PAPR reduction techniques is still open to researchers and many recent researches have been made in this field because so far, no technique has been chosen to be the best approach. For future works, this article offers sufficient knowledge regarding the theoretical aspects of the approaches so that a researcher can start with his/her work more easily. Moreover, the characteristics and comparisons provided in this article make it easier to choose which approach to work on based on the available hardware devices and the cost of implementations.

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**Data Availability Statement:** All of the data supporting the findings of the presented study are available from corresponding author on request.

**References**


بحث مراجعة: طرق المحدى من نسبة الذروة إلى متوسط القدرة في أنظمة OFDM

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الخلاصة

يتم هذا البحث بنظام تعدد الإرسال بتقسيم التردد التردد (OFDM) المعتمد في العديد من نظم الاتصال اللاسلكي لإرسال إشارات متعددة الموجات الترددية مع القدرة على مقاومة التداخل والضوضاء التي تواجه النظام. ولكن لاتزال هناك بعض المشاكل مهمة في نظام OFDM و من أهمها ارتفاع نسبة الذروة إلى متوسط القدرة (PAPR) وهي النسبة العالية لإشارات OFDM إلى تقسيم الذروة إلى متوسط القدرة PAPR في الأجهزة المستخدمة لنظام OFDM. تؤدي زيادة PAPR في الدراسة السابقة لبحث تقسيم الذروة إلى تقسيم PAPR و كذلك لها محسوس و مساواة مختلفة عن بعضها. الهدف من هذه المقالة هو استعراض و مراجعة الأساليب المختلفة لتقسيم PAPR ومقارنتها لاختيار أفضل تقسيم نظام معي.

الكميات الدائمة: نظام تعدد الإرسال بتقسيم التردد التعامد (OFDM) ؛ نسبة الذروة إلى متوسط القدرة (PAPR) ؛ القطع والتصفية ؛ المصاحبة ؛ التمدد.