Kirkuk University Journal /Scientific Studies (KUJSS)
Volume 13, Issue 4, December 2018 , pp. (34-44)
ISSN: 1992-0849 (Print), 2616-6801 (Online)

Generalized Weak Forms of Irresolute Mappings in Intuitionistic Topological Spaces

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Abstract

The purpose of this paper is to study a new classes of irresolute mappings called Intuitionistic Generalized Pre irresolute mappings, Intuitionistic Generalized Semi irresolute mappings, Intuitionistic Generalized α-irresolute mappings and Intuitionistic Generalized β-irresolute mappings with study of their properties. Then we investigate relationships between them.

Keywords: intuitionistic set, intuitionistic topological spaces, Irresolute mappings, Intuitionistic Generalized mappings.

DOI: http://doi.org/10.32894/kujss.2018.13.4.3

Kirkuk University Journal /Scientific Studies (KUJSS)
Volume 13, Issue 4, December 2018 , pp. (34-44)
ISSN: 1992-0849 (Print), 2616-6801 (Online)

تعميم الصيغ الضعيفة للتطبيقات المحيرة في الفضاءات التبولوجية الحدسية

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الملخص

في هذا البحث قدمنا صفوف جديدة من التطبيقات المحيرة وهي: التطبيق المحير من النوع Pre المعمم والتطبيق المحير شبه المعمم والتطبيق المحير من النوع α– المعمم والتطبيق من النوع β– المعمم مع دراسة خواصها. ثم استقصينا العلاقات فيما بين هذه المفاهيم.

الكلمات الدالة: المجموعة الحدسية، الفضاءات التبولوجية الحدسية، التطبيقات المحيرة، التطبيقات المعممة الحدسية.

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1. Introduction:

In 1965, Zadeh [8] introduced the concept of fuzzy set. later Atanassov introduced the concept of Intuitionistic fuzzy set [1,2] using generalized of fuzzy set, this concept used to defined intuitionistic fuzzy special sets and "intuitionistic fuzzy topological spaces which introduced by Coker [4,5]. Finally, in 2000, Coker [6] introduced the concept of intuitionistic topological spaces In this paper we introduce a new classes of weak irresolute mappings with study their properties. Finally, we investigate relationships between them.

2. Preliminaries:

Since We require the following known definitions, notations, and some properties so we recall them in this section.

Definition 2.1 [6] Let $\widetilde{M} \subseteq X \neq \emptyset$ **.** The Intuitionistic set $\widetilde{M}$ (IS, for short) is the form $\widetilde{M} = \langle x, M_1, M_2 \rangle$ and M_1 , $M_2 \subseteq X$ with condition $M_1 \cap M_2 = \emptyset$ **.** The set M_1 is a set of members of $\widetilde{M}$ and M_2 is a set of non-members of $\widetilde{M}$.

Definition 2. 2 [6] Let $X \neq \emptyset$, and let $\widetilde{M} = \langle x, M_1, M_2 \rangle$ and $\widetilde{N} = \langle x, N_1, N_2 \rangle$ are two IS. also, let$\{ \widetilde{M}_s ; s \in S\}$ be a collection of Intuitionistic sets in X , with $\widetilde{M}_i = \langle x, M_s^{(1)}, M_s^{(2)} \rangle$, where $M_s^{(1)}$ is a set of members of $\widetilde{M}$ and $M_s^{(2)}$ is a set of non-members of $\widetilde{M}$ .Then:

1) $\widetilde{M} \subseteq \widetilde{N}$ iff $M_1 \subseteq N_1$ and $N_2 \subseteq M_2$,

2) $\widetilde{M} = \widetilde{N}$ iff $\widetilde{M} \subseteq \widetilde{N}$ and $\widetilde{N} \subseteq \widetilde{M}$,

3) The complement of $\widetilde{M}$ is denoted by $\widetilde{M}^c$ and $\widetilde{M}^c = \langle x, M_2, M_1 \rangle$,

4) $\cup \widetilde{M}_i = \langle x, \cup M_s^{(1)}, \cap M_s^{(2)} \rangle$, $\cap \widetilde{M}_i = \langle x, \cap M_s^{(1)}, \cup M_s^{(2)} \rangle$,

5) $\dot{\emptyset} = \langle x, \emptyset, X \rangle$, $\dot{X} = \langle x, X, \emptyset \rangle$**.**

Definition 2.3 [4] Let $X \neq \emptyset$, $w \in X$ and let $\widetilde{M} = \langle x, M_1, M_2 \rangle$ be an Intuitionistic set the Intuitionistic point (IP , for short) IS $\dot{w}$ defined by $\dot{w} = \langle x, \{w\}, \{w\}^c \rangle$ in X **.** Also a Vanishing Intuitionistic point defined by IS $\ddot{w} = \langle x, \emptyset, \{w\}^c \rangle$ in X **.** The IS $\dot{w}$ belongs to $\widetilde{M}$ ($\dot{w} \in M$, for short) iff $w \in M_1$, also IS $\ddot{p}$ contained in $\widetilde{M}$ $(\ddot{w} \in \widetilde{M}$, for short$)$ iff $w \notin M_2$**.**

Definition 2.4 [3] Let $X \neq \emptyset$. An Intuitionistic topology (ITS, for short) on X is a collection μ of an Intuitionistic sets in X with the following conditions:

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1) $\dot{\emptyset}, \dot{X} \in \mu$.

2) μ is closed under finite intersections.

3) μ is closed under arbitrary unions

Each element in μ is called Intuitionistic open set denoted by " IOS ". The complement of an Intuitionistic open set is called Intuitionistic closed set Denoted by "ICS".

Definition 2.5 [6] Let X , Y $\neq \emptyset$, and $f : (X, \mu) \longrightarrow (Y, \gamma)$ be a mapping .

a) If $\tilde{N} = \langle y, N_1, N_2 \rangle$ is an IS in Y. Then the inverse image of $\tilde{N}$ defined by $f^{-1}(\tilde{N}) = \langle x, f^{-1}(N_1), f^{-1}(N_2) \rangle$.

b) If $\tilde{M} = \langle x, M_1, M_2 \rangle$ is an IS in X. Then $f(\tilde{V}) = \langle y, f(M_1), \breve{f}(M_2) \rangle$ is an IS in Y,X where $\breve{f}(\tilde{M}) = \overline{\left(f(\overline{M_2})\right)}$.

Definition 2.6 [6] Let (X, μ) be an ITS and let $\tilde{M} = \langle x, M_1, M_2 \rangle$ be IS . The Interior (namely, $int(\tilde{M})$) and the Closure (namely, $cl(\tilde{M})$) of a set $\tilde{M}$ defined by :

$int(\tilde{M}) = \cup\{ \tilde{J} : \tilde{J} \subseteq \tilde{M}, J \in \mu \}$,

$cl(\tilde{M}) = \cap\{ \tilde{K} : \tilde{M} \subseteq \tilde{K}, \overline{\tilde{K}} \in \mu\}$.

Also,

1- $sint(\tilde{M}) = \cup\{ \tilde{V} : \tilde{V} \subseteq \tilde{M}, V \in ISOX \}$,

$scl(\tilde{M}) = \cap\{ \tilde{J} : \tilde{M} \subseteq \tilde{J}, \overline{\tilde{J}} \in ISCS\}$.

2- $pint(\tilde{M}) = \cup\{ \tilde{V} : \tilde{V} \subseteq \tilde{M}, V \in IPOX \}$,

$pcl(\tilde{M}) = \cap\{ \tilde{J} : \tilde{M} \subseteq \tilde{J}, \overline{\tilde{J}} \in IPCS\}$.

3- $\alpha int(\tilde{M}) = \cup\{ \tilde{V} : \tilde{V} \subseteq \tilde{M}, V \in I\alpha OX \}$,

$\alpha cl(\tilde{M}) = \cap\{ \tilde{J} : \tilde{M} \subseteq \tilde{J}, \overline{\tilde{J}} \in I\alpha CS\}$.

4- $\beta int(\tilde{M}) = \cup\{ \tilde{V} : \tilde{V} \subseteq \tilde{M}, V \in I\beta OX \}$,

$\beta cl(\tilde{M}) = \cap\{ \tilde{J} : \tilde{M} \subseteq \tilde{J}, \overline{\tilde{J}} \in I\beta CS\}$.

Definition 2.9. [7] Let (X, μ) be an ITS. An intuitionistic set $\tilde{M}$ of X is said to be :

1) Intuitionistic generalizes semi-open set(IGSOS , for short) if $\forall$ U ISCS s.t $U \subseteq \tilde{M}$ then $U \subseteq int(\tilde{M})$. The complement of IGSOS is called IGSCS .

2) Intuitionistic generalizes pre-open set (IGPOS , for short) if $\forall$ U IPCS s.t

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$U \subseteq \widetilde{M}$ then $U \subseteq int(\widetilde{M})$. The complement of IGPOS is called IGPCS .

3) Intuitionistic generalizes α-open set (IGαOS , for short) if $\forall$ U IαCS s.t $U \subseteq \widetilde{M}$ then $U \subseteq int(\widetilde{M})$. The complement of IG$\alpha$OS is called IG$\alpha$CS .

4) Intuitionistic generalizes β-open set (IGβOS , for short) if $\forall$ U IβCS s.t $U \subseteq \widetilde{M}$ then $U \subseteq int(\widetilde{M})$. The complement of IG$\beta$OS is called IG$\beta$CS .

3. Intuitionistic Generalized Pre Irresolute & Semi Irresolute Mappings:

In this part we introduce Intuitionistic Generalized pre irresolute, Intuitionistic Generalized semi irresolute, Intuitionistic Generalized $\beta-$irresolute, Intuitionistic Generalized α-irresolute of mappings and study their properties.

Definition 3.1: A mapping $h: (Q, \mu) \rightarrow (W, \gamma)$ is an Intuitionistic Generalized Pre Irresolute (IGPIr, for short) (resp., Intuitionistic Generalized semi Irresolute (IGSIr, for short), Intuitionistic generalized α irresolute (IGαIr, for short) Intuitionistic Generalized β Irresolute (IGβIr, for short) mapping if $h^{-1}(\widetilde{M})$ is IGPCS (resp., is IGSCS, IGαCS, IGβCS) in (Q, μ) for every IGPCS $\widetilde{M}$ of (W, γ) (resp., for every IGSCS $\widetilde{M}$, IGαCS $\widetilde{M}$ IGβCS $\widetilde{M}$ of (W, γ)) .

Proposition 3.2: Let $h: (Q, \mu) \rightarrow (W, \gamma)$ and $k: (W, \gamma) \rightarrow (T, \delta)$ be IGPIr mapping. Then $k \circ h: (Q, \mu) \rightarrow (T, \delta)$ is an IGPIr mapping.

Proof: Let $\widetilde{M}$ be an IGPCS in T. Then $k^{-1}(\widetilde{M})$ is an IGPCS in W. Since h is an IGPIr, so that $h^{-1}(k^{-1}(\widetilde{M}))$ is an IGPCS in Q. Thus $k \circ h$ is IGPIr mapping.

Proposition 3.3: Let $h: (Q, \mu) \rightarrow (W, \gamma)$ and $k: (W, \gamma) \rightarrow (T, \delta)$ be IGαIr mapping. Then $k \circ h: (Q, \mu) \rightarrow (T, \delta)$ is an IGSIr mapping.

Proof: Let $\widetilde{M}$ be an IGαCS in T. Then $k^{-1}(\widetilde{M})$ is an IGαCS in W, since h is IGαIIr, and $\forall$ IGαCS is IGSCS. Thus $h^{-1}(k^{-1}(\widetilde{M}))$ is an IGSCS in Q. Therefore $k \circ h$ is IGSIr mapping.

Proposition 3.4: Let $h: (Q, \mu) \rightarrow (W, \gamma)$ be an IGPIr mapping and $k: (W, \gamma) \rightarrow (T, \delta)$ be IGP continuous mapping , then $k \circ h: (Q, \mu) \rightarrow (T, \delta)$ is IGP continuous mapping.

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Proof: Let $\widetilde{M}$ be ICS in T. Then $k^{-1}(\widetilde{M})$ is IGPCS in W. Since h is an IGPIr mapping, $h^{-1}(k^{-1}(\widetilde{M}))$ is IGPCS in Q. Therefore, $k \circ h$ is IGPS continuous mapping.

Proposition 3.5: Let $h: (Q, \mu) \to (W, \gamma)$ be IGαIr mapping and $k: (W, \gamma) \to (T, \delta)$ be IGα continuous mapping , then $k \circ h: (Q, \mu) \to (T, \delta)$ is IGS continuous mapping.

Proof: Let $\widetilde{M}$ be ICS in T .Then $k^{-1}(\widetilde{M})$ is an IGαCS in W. Since h is IGαIr mapping, $h^{-1}(k^{-1}(\widetilde{M}))$ is IGSCS in Q and ∀ IGαCS is IGSCS. Hence $k \circ h$ is IGS continuous mapping.

Proposition 3.6: Let $h: (Q, \mu) \to (W, \gamma)$ be IGPIr mapping and $k: (W, \gamma) \to (T, \delta)$ be IGP continuous mapping , then $k \circ h: (Q, \mu) \to (T, \delta)$ is IGβ continuous mapping.

Proof: from the definition.

Theorem 3.7: Let $h: (Q, \mu) \to (W, \gamma)$ be a mapping. Then the following conditions are equivalent:

(i) $h^{-1}(\widetilde{M})$ is IGPOS in Q ∀ IGPOS $\widetilde{M}$ in W, (ii) $h^{-1}\,Pint(\widetilde{M}) \subseteq Pint\, h^{-1}(\widetilde{M})$ for every IS $\widetilde{M}$ of W, (iii) $Pcl\, h^{-1}(\widetilde{M}) \subseteq h^{-1}\,Pcl(\widetilde{M})$ ∀ IS $\widetilde{M}$ of Y.

Proof: (i) ⇒ (ii) Let $\widetilde{M}$ be IS in W and $Pint(\widetilde{M}) \subseteq \widetilde{M}$. Also $h^{-1}pint(\widetilde{M}) \subseteq h^{-1}(\widetilde{M})$, since $Pint(\widetilde{M})$ is IPOS in W, so that IGPOS in W. Thus $h^{-1}\,Pint(\widetilde{M})$ is IGPOS in Q, and $h^{-1}\,Pint(\widetilde{M})$ is IPOS in Q . Hence $h^{-1}Pint(\widetilde{M}) = Pint\, h^{-1}\,Pint(\widetilde{M}) \subseteq Pint\, h^{-1}(\widetilde{M})$.

(ii) ⇒ (iii) is easy by taking complement in (ii).

(iii) ⇒ (i) Let $\widetilde{M}$ be an IGPCS in W. Since $\widetilde{M}$ is an IPCS in W and $Pcl(\widetilde{M}) = \widetilde{M}$. Hence $h^{-1}(\widetilde{M}) = h^{-1}\,Pcl(\widetilde{M}) \supseteq Pcl\, h^{-1}(\widetilde{M})$, by assumption . This implies $h^{-1}(\widetilde{M})$ is IGPCS in Q .

Theorem 3.8: Let $h: (Q, \mu) \to (W, \gamma)$ be a mapping. Then the following conditions are equivalent:

(i) $h^{-1}(\widetilde{M})$ is IGSOS in Q ∀ IGSOS $\widetilde{M}$ in W, (ii) $h^{-1}\,\alpha int(\widetilde{M}) \subseteq \alpha int\, h^{-1}(\widetilde{M})$ for every IS $\widetilde{M}$ of W, (iii) $Scl\, h^{-1}(\widetilde{M}) \subseteq h^{-1}\,Scl(\widetilde{M})$ ∀ IS $\widetilde{M}$ of Y.

Proof: (i) ⇒ (ii) Let $\widetilde{M}$ be any IS in W and $\alpha int(\widetilde{M}) \subseteq \widetilde{M}$. Also $h^{-1}\alpha int(\widetilde{M}) \subseteq h^{-1}(\widetilde{M})$. Since $\alpha int(\widetilde{M})$ is IαOS in W, so that IGSOS in W. Therefore $h^{-1}\,\alpha int(\widetilde{M})$ is IGSOS in Q, and $h^{-1}\,\alpha int(\widetilde{M})$ is ISOS in Q. Thus $h^{-1}\alpha int(\widetilde{M}) = \alpha int\, h^{-1}\,\alpha int(\widetilde{M}) \subseteq \alpha int\, h^{-1}(\widetilde{M})$.

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(ii) ⇒ (iii) is obvious.

(iii) ⇒ (i) Let $\widetilde{M}$ be an IGSCS in W. Since $\widetilde{M}$ is an ISCS in W and $Scl(\widetilde{M}) = \widetilde{M}$. Hence $h^{-1}(\widetilde{M}) = Scl\, h^{-1}(\widetilde{M}) \subseteq h^{-1}Scl(\widetilde{M})$,by assumption. Therefore $h^{-1}(\widetilde{M})$ is an IGSCS in Q.

Proposition 3.9: Let $h: (Q, \mu) \to (W, \gamma)$ and $k: (W, \gamma) \to (T, \delta)$ be IGαIr mapping. Then $k \circ h: (Q, \mu) \to (T, \delta)$ is IGP mapping.

Proof: Let $\widetilde{M}$ be IGSCS in T. Then $k^{-1}(\widetilde{M})$ is IGSCS in W. Since h is IGαIr, $h^{-1}(k^{-1}(\widetilde{M}))$ is IGαCS in Q , by assumption. Since every IGαCS is IGPCS. Thus $h^{-1}(k^{-1}(\widetilde{M}))$ is IGSCS in Q. Therefore $k \circ h$ is IGPIr mapping.

Proposition 3.10: Let $h: (Q, \mu) \to (W, \gamma)$ be IGPIr mapping and $k: (W, \gamma) \to (T, \delta)$ be IGα continuous mapping, then $k \circ h: (Q, \mu) \to (T, \delta)$ is IGP continuous mapping.

Proof: Let $\widetilde{M}$ be ICS in T. Then $k^{-1}(\widetilde{M})$ is IGPCS in W. Since h is an IGPIr mapping, $h^{-1}(k^{-1}(\widetilde{M}))$ is IGαCS in Q. Since every IGαCS is IGPCS. Thus $h^{-1}(k^{-1}(\widetilde{M}))$ is IGPCS in Q. Hence $k \circ h$ is IGP continuous mapping.

Proposition 3.11: Let $h: (Q, \mu) \to (W, \gamma)$ be IGαIr mapping and $k: (W, \gamma) \to (T, \delta)$ be IGα continuous mapping , then $k \circ h: (Q, \mu) \to (T, \delta)$ is IGS continuous mapping.

Proof: Let $\widetilde{M}$ be IαCS in T. Then $k^{-1}(\widetilde{M})$ is IGαCS in W. Since h is IGαIr mapping, $h^{-1}(k^{-1}(\widetilde{M}))$ is IGαCS in Q . Since every IGαCS is IGSCS. Thus $h^{-1}(k^{-1}(\widetilde{M}))$ is IGSCS in Q. Hence $k \circ h$ is IGS continuous mapping.

Theorem 3.12: Let $h: (Q, \mu) \to (W, \gamma)$ be a mapping. Then the following conditions are equivalent (i) $h^{-1}(\widetilde{M})$ is IGPOS in Q ∀ IGαOS $\widetilde{M}$ in W, (ii) $h^{-1}Pint(\widetilde{M}) \subseteq Pint\, h^{-1}(\widetilde{M})$ ∀ IS $\widetilde{M}$ of W, (iii) $Pcl\, h^{-1}(\widetilde{M}) \subseteq h^{-1} Pcl(\widetilde{M})$ ∀ IS $\widetilde{M}$ of W.

Proof: (i) ⇒ (ii) Let $\widetilde{M}$ be Is in W and $Pint(\widetilde{M}) \subseteq \widetilde{M}$. Also $h^{-1}Pint(\widetilde{M}) \subseteq h^{-1}(\widetilde{M})$. Since $Pint(\widetilde{M})$ is IPOS in W, so that IGαOS in W. Therefore $h^{-1}Pint(B)$ is an IGPOS in Q, and $h^{-1}Pint(B)$ is IPOS in Q. Hence $h^{-1}Pint(B) = Pint\, h^{-1}Pint(B) \subseteq Pint h^{-1}(B)$.
(ii) ⇒ (iii) is clear .

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(iii) ⇒ (i) Let $\tilde{M}$ be IGαCS in W. Since $\tilde{M}$ is IαCS in W and $\alpha cl(\tilde{M}) \subseteq Pcl(\tilde{M})$. Thus $Pcl\ h^{-1}(\tilde{M}) \subseteq h^{-1}\ Pcl(\tilde{M})$, by assumption. Therefore $h^{-1}(\tilde{M})$ is IGPCS in Q.

Proposition 3.13: Let $h: (Q, \mu) \rightarrow (W, \gamma)$ be IGSIr mapping and $k: (W, \gamma) \rightarrow (T, \delta)$ be IG continuous mapping , then $k \circ h: (Q, \mu) \rightarrow (T, \delta)$ is IGS continuous mapping.

Proof: Let $\tilde{M}$ be ICS in T. Then $k^{-1}(\tilde{M})$ is IGSCS in W. Since h is an IGSIr mapping, $h^{-1}(k^{-1}(\tilde{M}))$ is IGCS in Q. Since every IGCS is IGSCS. Thus $h^{-1}(k^{-1}(\tilde{M}))$ is IGSCS in Q. Hence $k \circ h$ is IGS continuous mapping.

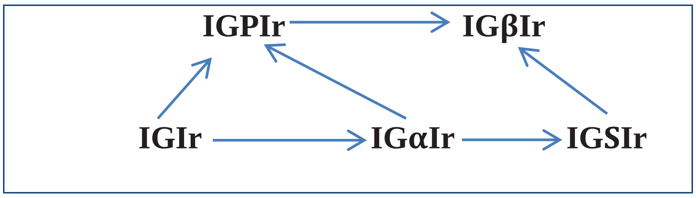
Proposition 3.14: Let $h: (Q, \mu) \rightarrow (W, \gamma)$ be IGPIr mapping and $k: (W, \gamma) \rightarrow (T, \delta)$ be IGp continuous mapping , then $k \circ h: (Q, \mu) \rightarrow (T, \delta)$ is IGβ continuous mapping.

Proof: Let $\tilde{M}$ be an ICS in T. Then $k^{-1}(\tilde{M})$ is IGPCS in W. Since h is IGPIr mapping, $h^{-1}(k^{-1}(\tilde{M}))$ is IGPCS in Q. Since every IGPCS is IGβSCS. Thus $h^{-1}(k^{-1}(\tilde{M}))$ is an IGβCS in Q. Hence $k \circ h$ is IGβ continuous mapping.

4. The Relations Among Intuitionistic Generalized Pre Irresolute, Intuitionistic Generalized β −Irresolute, Intuitionistic Generalized α −Irresolute & Semi Irresolute Mappings:

First, we give this proposition :

Proposition 4.1. The implication among some types of mappings are given by the following diagram.



Proof : IGPIr ⟶ IGβIr

Let $h: (Q, \mu) \rightarrow (W, \gamma)$ be a mapping. Let $\tilde{M}$ be IPCS in W. Since h is IGPIr then $h^{-1}(\tilde{M})$ is IGPCS in Q, since every IPC(W) is IβC(W). Thus $h^{-1}(\tilde{M})$ is IGβCS in Q ∀ $\tilde{M}$ be IβCS in W. Therefore h is IGβIr.

IGIr ⟶ IGαIr

Let $h: (Q, \mu) \rightarrow (W, \gamma)$ be a mapping. Let $\tilde{M}$ be ICS in W. Since h is IGIr then

$h^{-1}(\widetilde{M})$ is IGCS in Q, since every IC(W) is IαC(W) .Thus $h^{-1}(\widetilde{M})$ is IGαCS in Q $\forall$ $\widetilde{M}$ be IαCS in W. Therefore h is IGαIr .

IGIr $\longrightarrow$ IGPIr

Its obvious .

IGαIr $\longrightarrow$ IGSIr

Its obvious .

IGSIr $\longrightarrow$ IGβIr

Let $h: (Q, \mu) \rightarrow (W, \gamma)$ be a mapping. Let $\widetilde{M}$ be ISCS in W. Since h is IGSIr then $h^{-1}(\widetilde{M})$ is IGSCS in Q, since every ISC(W) is IβC(W). Thus $h^{-1}(\widetilde{M})$ is IGβCS in Q $\forall$ $\widetilde{M}$ be IβCS in W. Therefore h is IGβIr .

The reverse of above Proposition is not true, the next examples are show the cases.

Example 4.2. Let $Q = \{y, t, z\}$ with topology $\mu = \{\dot{Q}, \dot{\emptyset}, \widetilde{B}, \widetilde{H}, \widetilde{T}, \widetilde{E}\}$, where $\widetilde{B} = \langle q, \{y\}, \{t, z\}\rangle$, $\widetilde{H} = \langle q, \{y, z\}, \emptyset\rangle$, $\widetilde{T} = \langle q, \{y\}, \emptyset\rangle$, $\widetilde{E} = \langle q, \{y\}, \{z\}\rangle$, $W = \{2,3,4\}$ with topology $\gamma = \{\dot{W}, \dot{\emptyset}, \widetilde{K}, \widetilde{L}\}$, where $\widetilde{K} = \langle w, \{3\}, \{4,5\}\rangle$, $\widetilde{L} = \langle w, \{3,4\}, \emptyset\rangle$.

Define a mapping

$h: (Q, \mu) \rightarrow (W, \gamma)$ as $h(\{y\}) = \{2\}$, $h(\{t\}) = \{4\}, h(\{z\}) = \{3\}$. Then h is IGβIr, because $\forall$ $\widetilde{M}$ be IGβCS in W, $h^{-1}(\widetilde{M})$ is IGβCS in Q. But h is not IGPIr, because $h^{-1}(\{2,4\}) = \{y, t\}$ is not IGPCS in Q. Also h is not IGSIr , because $h^{-1}(\{3,4\}) = \{z, t\}$ is not IGSCS in Q.

Example 4.3. Let $Q = \{m, n, r\}$ with topology $\mu = \{\dot{Q}, \dot{\emptyset}, \widetilde{R}, \widetilde{S}\}$, where $\widetilde{R} = \langle q, \{m\}, \{n, r\}\rangle$, $\widetilde{S} = \langle q, \{m\}, \emptyset\rangle$, $W = \{5,6,7\}$ with topology $\gamma = \{\dot{Y}, \dot{\emptyset}, \widetilde{T}, \widetilde{D}\}$, where $\widetilde{T} = \langle w, \{5\}, \{7\}\rangle$, $\widetilde{D} = \langle w, \{5\}, \emptyset\rangle$. Define a mapping $h: (Q, \mu) \rightarrow (W, \gamma)$ as $h(\{m\}) = \{5\}$, $h(\{n\}) = \{7\}, h(\{r\}) = \{6\}$. Then h is IGPIr, because $\forall$ $\widetilde{M}$ be IGPCS in W, $h^{-1}(\widetilde{M})$ is IGPCS in Q. But h is not IGIr, Because $h^{-1}(\{5,7\}) = \{m, n\}$ is not IGCS in Q.

Example 4.4. Let $Q = \{v, t, r, e\}$ with topology $\mu = \{\dot{Q}, \dot{\emptyset}, \widetilde{N}, \widetilde{L}, \widetilde{S}, \widetilde{Z}\}$, where $\widetilde{N} = \langle q, \{v\}, \{t, r, e\}\rangle$, $\widetilde{L} = \langle q, \emptyset, \emptyset\rangle$,$\widetilde{S} = \langle q, \emptyset, \{t, r, e\}\rangle$,$\widetilde{Z} = \langle q, \{v\}, \emptyset\rangle$, $W = \{2,4,6\}$ with topology $\gamma = \{\dot{W}, \dot{\emptyset}, \widetilde{F}, \widetilde{H}\}$, where $\widetilde{F} = \langle w, \{2\}, \{4,6\}\rangle$, $\widetilde{H} = \langle w, \{2\}, \emptyset\rangle$. Define a mapping $h: (Q, \mu) \to (W, \gamma)$ as $h(\{v\}) = \{4\}$, $h(\{t\}) = h(\{r\}) = \{6\}$, $h(\{e\}) = \{2\}$. Then h is IGSIr, because $\forall\ \widetilde{M}$ be IGSCS in W, $h^{-1}(\widetilde{M})$ is IGSCS in Q. But h is not IGαIr, because $h^{-1}(\{2,6\}) = \{t, r, e\}$ is not IGαCS in Q.

Example 4.5. Let $Q = \{u, o\}$ with topology $\mu = \{\dot{Q}, \dot{\emptyset}, \tilde{J}, \tilde{L}\}$, where $\tilde{J} = \langle q, \{u\}, \{o\}\rangle$, $\tilde{L} = \langle q, \{u\}, \emptyset\rangle$, $W = \{1,2\}$ with topology $\gamma = \{\dot{Y}, \dot{\emptyset}, \widetilde{N}, \widetilde{T}\}$, where $\widetilde{N} = \langle w, \emptyset, \{1\}\rangle$, $\widetilde{T} = \langle w, \emptyset, \emptyset\rangle$. Define a mapping $h : (Q, \mu) \to (W, \gamma)$ as $h(\{u\}) = \{1\}$, $h(\{o\}) = \{2\}$. Then h is IGαIr, because $\forall\ \widetilde{M}$ be IGαCS in W, $f^{-1}(\widetilde{M})$ is IGαCS in Q. But h is not IGIr, because $h^{-1}(\{2\}) = \{o\}$ is not IGCS in Q.

Remark 4.6. IGPIr and IGβIr is independent notions .The following two examples show these two cases .

Example 4.7. Let $Q = \{d, e, f\}$ with topology $\mu = \{\dot{Q}, \dot{\emptyset}, \tilde{S}, \tilde{L}, \widetilde{M}, \widetilde{N}, \widetilde{P}\}$, where $\tilde{S} = \langle q, \{f\}, \{e, d\}\rangle$, $\tilde{L} = \langle q, \{f\}, \emptyset\rangle$, $\widetilde{M} = \langle q, \{f, d\}, \{e\}\rangle$, $\widetilde{N} = \langle q, \{f\}, \{e\}\rangle$, $\widetilde{P} = \langle q, \{f, d\}, \emptyset\rangle$ and $W = \{7,8,9\}$ with topology $\gamma = \{\dot{W}, \dot{\emptyset}, \widetilde{K}, \widetilde{Y}, \widetilde{T}, V, \widetilde{O}\}$, where $\widetilde{K} = \langle w, \{7\}, \{8,9\}\rangle$, $\widetilde{Y} = \langle w, \{7\}, \emptyset\rangle$, $\widetilde{T} = \langle w, \{7,8\}, \{9\}\rangle$, $\widetilde{V} = \langle w, \{7\}, \{9\}\rangle$, $\widetilde{O} = \langle w, \{7,8\}, \emptyset\rangle$.
Define a mapping $h : (Q, \mu) \to (W, \gamma)$ as $h(\{d\}) = \{8\}$, $h(\{e\}) = \{7\}$, $h(\{f\}) = \{9\}$. Then h is IGPIr ,because $\forall\ \widetilde{M}$ be IGPCS in W, $h^{-1}(\widetilde{M})$ is IGPCS in Q . But h is not IGSIr, because $h^{-1}(\{8,9\}) = \{d, f\}$ is not IGSCS in Q.

Example 4.8. Let $Q = \{m, n, l\}$ with topology $\mu = \{\dot{Q}, \dot{\emptyset}, \widetilde{A}, \widetilde{U}, \tilde{S}, \widetilde{V}, \tilde{Z}, \widetilde{K}, \widetilde{N}\}$, where $\widetilde{A} = \langle q, \{m\}, \{n, l\}\rangle$, $\widetilde{U} = \langle q, \{m\}, \{n\}\rangle$, $\tilde{S} = \langle q, \{m\}, \emptyset\rangle$, $\widetilde{V} = \langle q, \{m, n\}, \emptyset\rangle$, $\tilde{Z} = \langle q, \emptyset, \emptyset\rangle$, $\widetilde{K} = \langle q, \emptyset, \{n, l\}\rangle$, $\widetilde{N} = \langle q, \emptyset, \{n\}\rangle$ and $W = \{5,6,8\}$ with topology $\gamma = \{\dot{W}, \dot{\emptyset}, \widetilde{E}, \widetilde{F}, \widetilde{R}\}$, where $\widetilde{E} = \langle w, \{5\}, \{6,8\}\rangle$, $\widetilde{F} = \langle w, \{5,8\}, \emptyset\rangle$, $\widetilde{R} = \langle w, \{5\}, \emptyset\rangle$. Define a mapping $h : (Q, \mu) \to (W, \gamma)$ as $h(\{m\}) = \{5\}$, $h(\{n\}) = \{6\}$, $h(\{l\}) = \{8\}$. Then h is IGSIr, because $\forall\ \widetilde{M}$ be IGSCS in W, $h^{-1}(\widetilde{M})$ is IGSCS in Q . But h is not IGPIr, because $h^{-1}(\{6,8\}) = \{n, l\}$ is not IGPCS in Q .

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