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3a\ quasi M-θ-ii-continuous functions in bi-Supra topological spaces

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Abstract.

In this paper , we introduces a new definition in bi-supra topological space , called M-θ- ii- open and via this definition , we introduce a new types of functions called quasi M-θ-ii-continuous functions which unifies some weak forms of quasi θ-ii-continuous functions in bi-supra topological spaces and investigate their properties.

Key Words and Phrases bi-supra topological space , M-θ-ii-open set, M-θ-ii-closed set , quasi M-θ-ii-continuous.

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في الفضاءات ثنائية التبولوجي θ M- ii-حول الدوال شبه الضعيفة من النوع شبه الفوقية

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المستخلص:

في هذا البحث قدمنا تعريفا جديدا أسمينا ii-open في الفضاءات ثنائية التبولوجي شبه الفوقية ومن خلال هذا التعريف قدمنا نوع من الدوال شبه الضعيفة والتي من خلالها عملنا على توحيد بعض الأشكال للدوال θ M- ii-من النوع في الفضاءات التبولوجية شبه الفوقية وتحرينا بعض θ ii-الضعيفة من النوع خصائصها وطرق تكافؤها .

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1. Introduction

In 1987, Noiri and Popa introduced M-open set and M-closed set and investigated a new class of functions called quasi θ-continuous functions[8] , Maghrabi and Juhani introduced M-continuous function , pre-M-open function and pre-M-closed function [3] . In this paper by using M-θ-ii-open sets in [12] denoted another sets is M-θ-i-open in bi- supra topological spaces every M-θ-*ii*-open (resp. M-θ-*ii*-closed) set is M-θ-*i*-open (resp. M-θ-*i*-open) sets but the converse is not true[10] ,

, the quasi M-θ-ii -continuity is introduced and studied in bi-supra topological spaces (Let X be non-empty space , let $\mathcal{S}$o(X) be the set of all semi open subset of the space X , (for short $\mathcal{ST}$) and let $\mathcal{P}$o(X) be the set of all pre open subset of X(for short $\mathcal{PT}$), Then , we say that (X,$\mathcal{ST}$,$\mathcal{PT}$) is a bi-supra topological space, [5] Moreover, basic properties of quasi M-θ-ii -continuous functions are investigated , also, relationships between quasi M-θ- ii-continuous functions and graphs are investigated.

2.Preliminaries.

Throughout this paper (X ,$\mathcal{T}_x$) and (Y, $\mathcal{T}_y$) (Simply, X and Y) represent topological spaces on which no separation axioms are assumed, unless otherwise mentioned. The closure , the interior and the complement a subset of A is denoted by cl(A), int(A) and(A^c or X\A) respectively . A subset A of a space X is said to be regular open[7] if it is the interior of its closure, i.e,(A = int(cl(A)). The complement of a regular- open set is referred to as a regular –closed set. A union of regular-open sets is called δ-open [7] The complement of a δ-open set is a δ-closed set. A subset A of a space(X, $\mathcal{T}_x$)is a θ-open set [9] if there exists an open set U containing x such that U⊆cl(U) ⊆ A. The set of all θ-interior points of A is said to be the θ-interior set and denoted by θ-int (A). A subset A of X is θ-open if A = θ- int (A) . The family of all θ- open sets of a space X is a topology on $\mathcal{T}_x$. The union of all θ-open (resp. δ-open) sets contained

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in A is called the θ-interior (resp. δ-interior) of A and it is denoted by θ- int(A) (resp. δ-int (A)). The intersection of all θ-closed (resp. δ- closed,) sets containing A is called the θ- closure (resp. δ- closure [4]) of A and it is denoted by θ- cl(A) (resp. δ-cl(A)).

We recall the following definitions and results, which are useful in the sequel

Definition 2.1[8] Let (X ,$\mathcal{T}_x$) be a topological space. Then a subset A of X is said to be:

(i) an M-open set, if A ⊆ cl(int_θ (A)) ∪ int(cl_δ(A)),

(ii) an M-closed set if int(cl_θ(A)) ∩ cl(int_δ(A)) ⊆A.

Definition 2.2[8] Let (X ,$\mathcal{T}_x$) be a topological space and A ⊆ X. Then:

(i) the M-interior of A is the union of all M-open sets contained in A and is denoted by M-int(A),

(ii) the M-closure of A is the intersection of all M-closed sets containing A and is denoted by M-cl(A) .

Definition 2.3[3] A function f : (X,$\mathcal{T}_x$)→ (Y , $\mathcal{T}_y$) is said to be:

(i) M-continuous [1] if f^{-1} (U) is M-open in X , for each U is $\mathcal{T}_y$

(ii) pre-M-open [2] if, f (U) is M-open in Y , for each U is M- open in X.

(iii) pre-M-closed [2] if, f (U) is M -closed in Y , for each U is M- closed in X.

Definition 2.4[11] A function f: (X,$\mathcal{ST}_x$,$\mathcal{PT}_x$) ⟶(Y,$\mathcal{ST}_y$,$\mathcal{PT}_y$) is called a quasi- θ-continuous function if f^{-1}(V) is a θ-open set in X for every θ-open set V of Y .

3 . A new type of bi-supra topological *space*

Definitions 3.1 Let (X,$\mathcal{ST}_x$, $\mathcal{PT}_x$) be a bi-supra topological space, and let G be a subset of X. Then G is said to be an ii- open set if $G=(AUB)\ U\ \emptyset$ where $A\in\mathcal{ST}$, $B\in\mathcal{PT}$ such that $A\notin \mathcal{PT}$, $A\cap B \neq \emptyset$. The Complement of ii- open set is called ii-closed set.

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Definitions 3.2_A subset A of a space (X,$\mathcal{ST}_x$, $\mathcal{PT}_x$) is called θ-ii-open set if there exists an ii- open set U containing x such that U ⊆ii- cl(U) ⊆ A.

Definitions 3.3 The set of all θ-ii-interior points of A is said to be the θ-ii-interior set and denoted by θ-ii-int (A). so , a subset A of X is θ-ii-open if and only if A = θ-ii- int (A).

Definitions 3.4_A union of regular-ii-open sets is called δ-ii-open . The complement of a δ-ii-open set is a δ-ii-closed set .

Definitions 3.5 A subset A of a bi-supra topological space (X,$\mathcal{ST}_x$, $\mathcal{PT}_x$)is called an M-ii-open set if A ⊆ ii- cl(θ-ii int(A)) ∪ ii- int(δ –ii-cl(A)). The union of all M-ii-open (resp. δ-ii-open,) sets contained in A is called the M-ii-interior (resp. δ-ii-interior) of A and it is denoted by M-ii-int(A) (resp . δ-ii-int (A)). The intersection of all M-ii-closed (resp. δ-ii- closed,) sets containing A is called the M-ii-closure (resp. δ-ii- closure) of A and it is denoted by M-ii-cl(A) (resp. δ-ii-cl(A)).

Definition 3.6_ A subset A of a space (X,$\mathcal{ST}_x$, $\mathcal{PT}_x$) is an M-θ-ii- open set if and only if for each x ∈ A there exists an M-ii-open set in X such that M-ii-cl(G) ⊆A.

Remark 3.7_Every M-θ-ii- open (resp. M-θ-ii- closed) set is an M-ii- open (resp. M-ii- closed) set , and every θ-ii- open (resp. θ-ii- closed) set is M-ii- open (resp. M-ii-closed) but the converse is not true as in that example

The implication between some types of sets are given by the following diagram

M-θ-ii- open ⟶ M-ii- open ⟵ θ-ii- open

Example 3.8._Let X = {a, b, c} and $\mathcal{T}_x$ = { ∅, {a}, {b}, {a,b}, X} . Then { ∅,{b, c}, {a,c}, {c}, X} , $\mathcal{ST}_x$= { ∅, {a}, {b},{a,b},{a,c}, {b,c}, X}= $\mathcal{T}_x^{\,c}$

$\mathcal{PT}_x$ = { ∅, {a},{b}, {a, b} , X}

ii-open in X = { ∅, {a, c}, {b, c}, X}, ii-closed in X = { ∅,{b} ,{a}, X}

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= { ∅, X} , in X θ-ii- open

= { ∅, X}, in X θ-ii-closed

Regular –ii-open in X= { ∅, X},

δ-ii-open in X= { ∅, X},

δ-ii-closed in X= { ∅, X},

M-ii-open in X= { ∅, {a},{b},{c}, {a, b} ,{a,c}, {b, c}, X},

M-ii-closed in X = { ∅, {b, c} {a, c},{a,b},{c},{b},{a} ,X},

M-θ-ii- open in X= { ∅, {a, b},{a,c}, {b, c} ,X} , so the sets {a},{b},{c} are M-ii-open sets but not M-θ-ii-open sets in X and not θ-ii- open in X .

Definition 3.9 A subset A in a bi-supra topological space $(X,\mathcal{ST}_x , \mathcal{PT}_x)$ is called M-θ-ii-closure and denoted by M-θ-ii-cl(A) and is defined to be the set of all points x of X such that for each an M-θ-ii- open set in X, M-θ-ii-cl(G) ∩ A ≠ ∅.

Definition 3.10 A subset A in bi-supra topological space $(X,\mathcal{ST}_x , \mathcal{PT}_x)$ is said to be M-θ-ii-closed if M-θ-ii-cl(A) = A. The complement of a M-θ-ii- closed set is an M-θ-ii-open set.

Lemma 3.11 For a subset A of a topological space $(X,\mathcal{T}_x)$ (resp.bi-supra topological space $(X,\mathcal{ST}_x ,\mathcal{PT}_x)$ the following statements are hold:

(i) If A ⊆ Fi, Fi is an M-closed [2](resp, M-ii-closed)set in X, then A ⊆ M-cl(A) ⊆ Fi,(resp.M -ii-cl(A) ⊆ Fi).

(ii) If Gi ⊆ A, Gi is an M-open[3] (resp . M-ii-open) set in X, then Gi ⊆ M-int(A) ⊆ A(resp. Gi ⊆ M-ii-int(A) ⊆ A) .

(iii) A is M-ii-closed in U if M-ii-d(A) ⊆ A

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(iv) M -ii-cl(A) = A ∪ M-ii-d(A) .

The set of θ-boundary (resp. θ-ii-boundary ,M-ii-boundary, M-ii-border) of A is denoted by θ-F_r (A) (resp. θ-ii-F_r (A),M-ii-F_r (A), M-ii- b(A)).

Proposition 3.12 Let A be a subset of (X,$\mathcal{T}_x$) a topological space (resp. (X,$\mathcal{ST}_x$,$\mathcal{PT}_x$) a bi-supra topological space) . Then , the following statements are hold:

(i) θ-F_r (A) = θ-cl (A) \ θ-int (A))[6] (resp. θ-ii-F_r (A) = θ-ii cl (A) \(θ-ii int (A))

(ii) M-F_r (A) = M-cl(A) \ M-int(A) [4] (resp.M-ii-F_r (A) = M-ii-cl(A) \(M-ii-int(A))

(iii) M- b(A) = A\M-int(A) [2] (resp. M-ii- b(A) = A\M-ii-int(A)).

We recall the following definitions and results, which are useful in the sequel:

Definition 3.13 A function *f*: (X,$\mathcal{ST}_x$,$\mathcal{PT}_x$) ⟶ (Y,$\mathcal{ST}_y$,$\mathcal{PT}_y$) is called M-*ii*-continuous if f^{-1}(V) is M-*ii*-open in X for every *ii*-open set V inY .

Example 3.14

Let X = {a, b, c} and $\mathcal{T}_x$ = { ∅, {a}, {b}, {a,b}, X} . Then

{ ∅,{b, c}, {a,c}, {c}, X} , $\mathcal{ST}_x$= { ∅, {a}, {b},{a,b},{a,c}, {b,c}, X}= $\mathcal{T}_x{}^c$

$\mathcal{PT}_x$ = { ∅, {a},{b}, {a, b} , X}

ii-open in X = { ∅, {a, c}, {b, c}, X}, ii-closed in X = { ∅,{b} ,{a}, X}

M-ii-open in X= { ∅, {a},{b},{c}, {a, b} ,{a,c}, {b, c}, X}

M-ii-closed in X = { ∅, {b, c} {a, c},{a,b},{c},{b},{a} ,X}

New, let Y={a,b,c} and , $\boldsymbol{\mathcal{T}}_{\mathbf{y}}$={∅,{a},{c},{a,c},Y} , $\boldsymbol{\mathcal{T}}_{\mathbf{y}}{}^{\mathbf{c}}$={Y,{b,c},{a,b},{b},∅}. Then

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$\boldsymbol{\mathcal{ST}}_{\mathbf{y}}$= { ∅, {a}, {c},{a,c},{a,b}, {b,c}, Y} , $\boldsymbol{\mathcal{PT}}_{\mathbf{y}}$ = { ∅, {a},{c}, {a, c} , Y}

ii-open in Y ={ ∅,{a,b},{b,c},Y}, *ii*-closed in Y={∅,{c},{a},Y} .

If *f*: (X,$\boldsymbol{\mathcal{ST}}_{\mathbf{x}}$,$\boldsymbol{\mathcal{PT}}_{\mathbf{x}}$) ⟶(Y,$\boldsymbol{\mathcal{ST}}_{\mathbf{y}}$,$\boldsymbol{\mathcal{PT}}_{\mathbf{y}}$) is defined by *f* (a)=b , *f* (b)=a ,and *f* (c)=c then *f* is M-*ii*-continuous .

Theorem 3.15 Let (X,$\boldsymbol{\mathcal{ST}}_{\mathbf{x}}$,$\boldsymbol{\mathcal{PT}}_{\mathbf{x}}$) and (Y,$\boldsymbol{\mathcal{ST}}_{\mathbf{y}}$,$\boldsymbol{\mathcal{PT}}_{\mathbf{y}}$) be two bi-supra topological spaces and *f*:(X,$\boldsymbol{\mathcal{ST}}_{\mathbf{x}}$,$\boldsymbol{\mathcal{PT}}_{\mathbf{x}}$)⟶(Y,$\boldsymbol{\mathcal{ST}}_{\mathbf{y}}$,$\boldsymbol{\mathcal{PT}}_{\mathbf{y}}$) be a function . Then the following statements are equivalent :

(i) *f* is M-*ii*-continuous
(ii) M-*ii*-cl($\boldsymbol{f^{-1}}$(B))⊆ $\boldsymbol{f^{-1}}$(*ii*-cl(B)) for each *B* ⊆ Y
(iii) *f*(M-*ii*-cl(*A*)) ⊆*ii*-cl(*f*(*A*)) for each *A* ⊆ X
(iv) $\boldsymbol{f^{-1}}$(*ii*-int(B)) ⊆M-*ii*-int($\boldsymbol{f^{-1}}$(*ii*-b(B)) for each *B* =Y

Proof: (i) ⟶ (ii) Since B ⊆ *ii*-cl(B) ⊆Y which and is an *ii*-closed set. ,then by hypothesis, $\boldsymbol{f^{-1}}$(*ii*-cl(B)) is M-*ii*-closed in X. Hence, by **Lemma 3.11**, M-*ii*-cl($\boldsymbol{f^{-1}}$(B))⊆ $\boldsymbol{f^{-1}}$(*ii*-cl(B)) for each *B* ⊆ Y

(ii) ⟶ (iii) Let A ⊆ X. Then $\boldsymbol{f}$ (A) ⊆ Y, hence by hypothesis, M-ii-cl(A) ⊆ M-ii-cl($\boldsymbol{f^{-1}}$ ($\boldsymbol{f}$ (A))) ⊆ $\boldsymbol{f^{-1}}$ (ii-cl(f(A))). Therefore, $\boldsymbol{f}$ (M-ii-cl(A)) ⊆ *f* $\boldsymbol{f^{-1}}$ (ii-cl($\boldsymbol{f}$ (A))) ⊆ ii-cl($\boldsymbol{f}$ (A)), (iii)⟶ (i)Let V ⊆ Y be an ii-closed set. Then, $\boldsymbol{f^{-1}}$ (V) ⊆X. Hence, by (iii), $\boldsymbol{f}$ (M-ii-cl($\boldsymbol{f^{-1}}$ (V))) ⊆ ii-cl($\boldsymbol{f}$ ($\boldsymbol{f^{-1}}$ (V))) ⊆ ii-cl(V) = V. Thus M-ii-cl($\boldsymbol{f^{-1}}$ (V)) ⊆ $\boldsymbol{f^{-1}}$ (V) and hence $\boldsymbol{f^{-1}}$ (V) is M-ii-closed in X. Hence, $\boldsymbol{f}$ is M-ii-continuous, (iv)⟶ (i)Let U = Y be an ii-open set. Then by assumption, $\boldsymbol{f^{-1}}$ (U) = $\boldsymbol{f^{-1}}$ (ii-int(U)) ⊆ M-ii-int($\boldsymbol{f^{-1}}$ (U)). Hence, $\boldsymbol{f^{-1}}$ (U) is M-ii-open in X. Therefore, $\boldsymbol{f}$ is M-ii-continuous.

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Corollary 3.16 Let (X,$\mathcal{ST}_x$,$\mathcal{PT}_x$) and (Y,$\mathcal{ST}_y$,$\mathcal{PT}_y$) be two bi-supra topological spaces and f:(X,$\mathcal{ST}_x$,$\mathcal{PT}_x$)⟶(Y,$\mathcal{ST}_y$,$\mathcal{PT}_y$) be a function

(i) If f is M-ii-continuous , then M-*ii*-b(f^{-1}(B))) ⊆ f^{-1}(*ii*-b(B)) for each $B \subseteq$ Y

(ii) If f is M-ii-continuous , then M-*ii*-Fr(f^{-1}(B))) ⊆ f^{-1}(*ii*-Fr(B)) for each $B \subseteq$ Y

Theorem 3.16 Let (X,$\mathcal{ST}_x$,$\mathcal{PT}_x$) and (Y,$\mathcal{ST}_y$,$\mathcal{PT}_y$) be two bi-supra topological spaces and f:(X,$\mathcal{ST}_x$,$\mathcal{PT}_x$)⟶(Y,$\mathcal{ST}_y$,$\mathcal{PT}_y$) be a function . Then the following statement are equivalent :

(i) f is M-*ii*-continuous

(ii) f(M-*ii*-d(A)) ⊆*ii*-cl(f(A)) for each $A \subseteq$ X , where M-*ii*-d(A) is (The set of all M-*ii*-limit points of A is called M-*ii*-derived set of A and denoted by M-*ii*-*d(A)*).

Proof: (i) ⟶(ii) since f is M-*ii*-continuous then by theorem (3.15(iii)) f(M-*ii*-cl(A)) ⊆*ii*-cl(f(A)) for each $A \subseteq$ X , so f(M-*ii*-d(A))⊆ f(M-*ii*-cl(A)) ⊆*ii*-cl(f(A)) .

(ii) ⟶ (i) Let U be an *ii*-closed subset of Y . Then f^{-1}(U) ⊆ X hence by hypothesis f(M-*ii*-cl(f^{-1}(U))) ⊆*ii*-cl(f(f^{-1}(U))) ⊆*ii*-cl(U) =U . Therefore by lemma (3.11(iii),(iv))

Thus M-*ii*-cl(f^{-1}(U))= f^{-1}(U) ∪ M-*ii*-d(f^{-1}(U)) ⊆ f^{-1}(U) ∪ f^{-1}(U) = f^{-1}(U). Hence f^{-1}(U)= M-*ii*-cl(f^{-1}(U)) which is M-*ii*-closed set in X therefore f is M-*ii*-continuous .

Definition 3.17 A function f: (X,$\mathcal{ST}_x$,$\mathcal{PT}_x$) ⟶(Y,$\mathcal{ST}_y$,$\mathcal{PT}_y$) is called an M-θ-*ii*-continuous function , if for each x in X and each *ii*-open set V in Y containing $f(x)$, there exists an M-θ-*ii*-open set U in X containing x such that f(U) ⊆ V .

Remark 3.18 Every M-θ-*ii*-continuous function is M-*ii*-continuous but the converse is not true.

Proof:_ Directly from that definitions .

Example3.19

Let X={a,b,c}

$\mathcal{T}_x$ = { ∅, {a}, {b}, {a,b}, X} .Then

{ ∅,{b, c}, {a,c}, {c}, X} , $\mathcal{ST}_x$= { ∅, {a}, {b},{a,b},{a,c}, {b,c}, X}= $\mathcal{T}_x{}^c$

$\mathcal{PT}_x$ = { ∅, {a},{b}, {a, b} , X}

M-θ-ii-open in X= { ∅, {a},{b},{c}, {a, b} ,{a,c}, {b, c}, X}

M-θ-ii-closed in X = { ∅ , {b, c} {a, c},{a,b},{c},{b},{a} ,X}

And let let Y={a,b,c}

$\mathcal{T}_y$={∅,{a},{b},{a,b},Y} , $\mathcal{T}_y{}^c$={Y,{b,c},{a,c},{c},∅}

$\mathcal{ST}_y$= { ∅, {a}, {b},{a,b},{a,c}, {b,c}, Y} , $\mathcal{PT}_y$ = {∅, {a},{b}, {a, b} , Y}

ii-open in Y ={ ∅,{a,c},{b,c},Y}

ii-closed in Y={∅,{b},{a},Y}

If f : (X,$\mathcal{ST}_x$,$\mathcal{PT}_x$) ⟶(Y,$\mathcal{ST}_y$,$\mathcal{PT}_y$) is defined by *f* (a)=a , *f* (b)=c , and *f* (c)=b . Then *f* is M-θ-*ii*-continuous .

4. qusai M-θ-ii –continuous functions.

In this section , we introduces the concept of " qusai M-θ-ii –continuous functions" and some examples with many properties of this concept .

Definition 4.1 A function *f:* (X,$\mathcal{ST}_x$,$\mathcal{PT}_x$) ⟶(Y,$\mathcal{ST}_y$,$\mathcal{PT}_y$) is called quasi-θ-*ii*-continuous function if f^{-1}(V) is θ-*ii*-open set in X for every θ-*ii*-open set V of Y .

Definition 4.2 A function *f:* (X,$\mathcal{ST}_x$,$\mathcal{PT}_x$) ⟶(Y,$\mathcal{ST}_y$,$\mathcal{PT}_y$) is called a quasi-M-θ-*ii*-continuous function , if f^{-1}(V) is an M-θ-*ii*-open set in X for every θ-*ii*-open set V of Y .

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Example 4.3 See example (3.13) is holding definition

Proposition 4.4 Every quasi-M-$\boldsymbol{\theta}$-*ii*-continuous function is an M- $\boldsymbol{\theta}$-*ii*-continuous function .

Theorem 4.5 For a function : $(X,\mathcal{ST}_x,\mathcal{PT}_x) \longrightarrow (Y,\mathcal{ST}_y,\mathcal{PT}_y)$, the following statements are equivalent:

(i) $\boldsymbol{f}$ is quasi M- θ-ii-continuous,

(ii) For each x∈ X and each θ-ii-open V set in Y contains $\boldsymbol{f(x)}$, there exists an M-θ-ii-open set U in X contains x such that f(U)⊆ V,

(iii) $\boldsymbol{f^{-1}}$((F) is M- θ-ii-closed in X, for every θ-ii-closed set F of Y,

(iv) M- θ-ii-cl($\boldsymbol{f^{-1}}$((B)) ⊆ $\boldsymbol{f^{-1}}$((θ-ii-cl(B)), for each B ⊆ Y,

(v)$\boldsymbol{f}$ (M- θ-ii-cl(A)) ⊆ θ-ii-cl($\boldsymbol{f}$ (A)), for each A ⊆ X,

(vi) $\boldsymbol{f^{-1}}$((θ-ii-int(B)) ⊆ M- θ-ii-int($\boldsymbol{f^{-1}}$((B)), for each B ⊆ Y,

(vii) M- θ-ii- Fr ($\boldsymbol{f^{-1}}$((B)) ⊆ $\boldsymbol{f^{-1}}$((θ-ii- Fr (B)) , for each B ⊆ Y,

(viii) M- θ-ii-b($\boldsymbol{f^{-1}}$((B)) ⊆ $\boldsymbol{f^{-1}}$((θ-ii-b(B)), for each B ⊆ Y.

Proof. (i)⟶(ii). Let x∈ X and V ⊆ Y be a θ-ii-open set containing $\boldsymbol{f}$ (x). Then x∈ $\boldsymbol{f^{-1}}$((V). Hence by hypothesis, $\boldsymbol{f^{-1}}$((V) is M- θ-ii-open set of X containing x. We put U = $\boldsymbol{f^{-1}}$((V), then x∈ U and $\boldsymbol{f}$ (U) ⊆ V.

(ii)⟶(iii). Let F ⊆ Y be θ-ii-closed. Then Y\F is θ-ii-open if x∈ $\boldsymbol{f^{-1}}$ (Y\F), then $\boldsymbol{f}$ (x) ∈ Y\F. Hence by hypothesis, there exists an M- θ-ii-open set U containing x such that f(U) ⊆ Y\F, this implies that, x∈ U ⊆ $\boldsymbol{f^{-1}}$ (Y\F). Therefore, $\boldsymbol{f^{-1}}$ (Y\F) = ∪x∈{ U : $\boldsymbol{f^{-1}}$ (Y\F)} which is M- θ-ii-open in X. Therefore, $\boldsymbol{f^{-1}}$ (F) is M- θ-ii-closed.

(iii)⟶(i). Let V ⊆ Y be a θ-ii-open set. Then Y\V is θ-ii-closed in Y. By hypothesis, $\boldsymbol{f^{-1}}$ (Y\V) = X\$\boldsymbol{f^{-1}}$ (V) is M- θ-ii-closed and hence $\boldsymbol{f^{-1}}$ (V) is M-θ-ii-open. Therefore, $\boldsymbol{f}$ is quasi M- θ-ii-continuous.

(i)→ (iv). If B ⊆ X then θ-ii-cl(B) is θ-ii-closed , then by hypothesis, $\boldsymbol{f}^{-1}$ (θ-ii-cl(B)) is M- θ-ii-closed in X . Hence, by Lemma 3.11, M- θ-ii-cl($\boldsymbol{f}^{-1}$ (B)) ⊆ $\boldsymbol{f}^{-1}$ (θ-ii-cl(B)) for each B ⊆ Y.

(iv) →(v). Let A ⊆ X. Then $\boldsymbol{f}$ (A) ⊆Y, hence by hypothesis, M- θ-ii-cl(A) ⊆ M- θ-ii-cl($\boldsymbol{f}^{-1}$ ($\boldsymbol{f}$ (A))) ⊆ $\boldsymbol{f}^{-1}$ (θ-ii-cl($\boldsymbol{f}$ (A))). Therefore, $\boldsymbol{f}$ (M- θ-ii-cl(A)) ⊆ $\boldsymbol{f}$ $\boldsymbol{f}^{-1}$ (θ-ii-cl($\boldsymbol{f}$ (A))) ⊆ θ-ii-cl($\boldsymbol{f}$ (A)),

(v) →(i). Let V ⊆ Y be a θ-ii-closed set. Then, $\boldsymbol{f}^{-1}$ (V) ⊆ X . Hence, by hypothesis, f(M- θ-ii-cl($\boldsymbol{f}^{-1}$ (V))) ⊆ θ-ii-cl(f(f^{-1} (V))) ⊆ θ-ii-cl(V) = V. Thus M- θ-ii-cl(f^{-1} (V)) ⊆ f^{-1} (V) and hence f^{-1} (V) ∈ M- θ-ii-closed in X. Hence, f is quasi M-θ-ii-continuous,

(i) → (vi). ∀ B ⊆ X then θ-ii-int(B) is θ-ii-open , then by hypothesis, f^{-1} (θ-ii-int(B)) is an M- θ-ii-open set in X. Hence, by Lemma 3.11 , f^{-1} (θ-ii-int(B)) ⊆ M- θ-ii-int(f^{-1} (B)), for each B ⊆ Y.

(vi) → (i). Let V ⊆ Y be a θ-ii-open set. Then by assumption, f^{-1} (V) = f^{-1} (θ-ii-int(V)) ⊆ M- θ-ii-int(f^{-1} (V)) . Hence, f^{-1} (V) is M- θ-ii-open in X. Therefore, f is quasi M- θ-ii-continuous.

(vi) → (vii). Let V ⊆ Y. Then by hypothesis, f^{-1} (θ-ii-int(V)) ⊆ M- θ-ii-int(f^{-1} (V)) and so f^{-1} (V) \ M- θ-ii-int(f^{-1} (V)) ⊆ f^{-1} (V) \ f^{-1} (θ-ii-int(V)) = f^{-1} (V\ θ-ii-int(V)). By **Proposition 3.12**, M- θ-ii-Fr(f^{-1} (V)) ⊆ f^{-1} (θ-ii- Fr (V)).

Proposition 4.6 If f : (X,$\mathcal{ST}_x$,$\mathcal{PT}_x$) ⟶ (Y,$\mathcal{ST}_y$,$\mathcal{PT}_y$) is quasi-M- θ-ii-continuous and g: (Y,$\mathcal{ST}_y$,$\mathcal{PT}_y$) ⟶(Z,$\mathcal{ST}_z$,$\mathcal{PT}_z$) is quasi- θ-ii-continuous then gof is quasi-M- θ-ii-continuous .

Proof: let V⊆ Z be a θ-ii-open set and g be a quasi- θ-ii-continuous function . Then g^{-1}(V) is θ-ii-open in Y . But f is a quasi-M- θ-ii-continuous function , then $(gof)^{-1}$(V) is an M-θ-ii-open set in X , Hence gof is an M-θ-ii-continuous function

References

[1] A.I. EL-Maghrabi and M.A. AL-Juhani, Further properties on M-continuity, Proc. Math. Soc. Egypt, 22(2014), 63- 69

[2] A.I. EL-Maghrabi and M.A. AL-Juhani, M-open sets in topological spaces, Pioneer J. Math. Sciences, 4 (201 1), no. 2, 213-230.

[3] A.I.EL-Maghrabi and M.A.AL-Juhani , New weak forms of faint continuity, IJISET - International Journal of Innovative Science, Engineering & Technology, Vol. 1 Issue 4, 6 ,285 – 295 , (2014)

[4] E. Ekici, On e-open sets, DP*-sets and DPE*-sets and decompositions of continuity , Arabian J. Sci. Eng., 33, no. 2A, 269 – 282, (2008)

[5] Firas N. AL-Zandi , A new type of bi-supra topological spaces , MSC , University of Tikrit ,Colleg of Education Sciences Pure . Thesis in, A.D , 2014

[6] M. E. Abd El-Monsef, El-Deeb S. N. and R. A. Mahmoud, β-Open sets and β-continuous mappings, Bul l. Fac. Sci. Assiut Univ., 12, 77–90 , (1983)

[7] M.H. Stone, Application of the theory of Boolean rings to general topology, Tams. , 41, 375 – 381, (1937)

[8] Noiri, T., and Popa, V. (1987) Weak forms of faint continuity, Bull. Math. Soc. Sci. Math. R.S. Roumanie, 31(79), 163-168

[9] P.E.LongandL.Herrington,TheTθ-topology and faintly continuous functions, Kyungpook Math. J., 22(1982), 7—14

[10] Reem O.Rasheed, On θ- Continuity In Bi-Supra Topological Spaces, MSC , University of Tikrit ,Colleg of Education Sciences Pure . Thesis in, A.D , 2015

[11] S. Jafari, Some properties of quasi θ-continuous functions, Far East J. Math. Sci., 6(1998), no. 5, 689-696.

[12] Taha H. Jasim and Reem O. Rasheed, On strongly faintly M-θ-i-continuous functions in Bi-Supra Topological Space , Tikrit Journal of Pure Science 20 (4) 2015 , ISSN: 1813 – 1662.